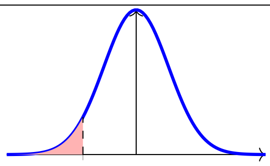
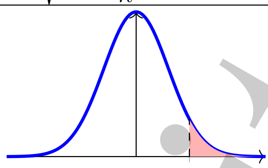
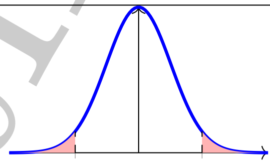
Hypothesis Testing Summary

ONE-PROPORTION Z-TEST (Normal approximation)

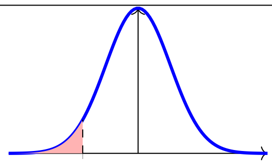
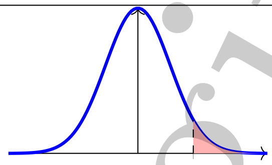
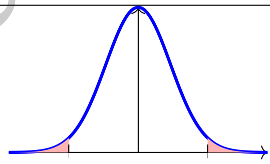
	Left-tailed	**Right-tailed**	**Two-tailed**
Assumptions	• Observations are independent • Data should follow a binomial distribution • Large sample: $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$		
Hypotheses	$H_0 : p \geq p_0$ $H_1 : p < p_0$	$H_0 : p \leq p_0$ $H_1 : p > p_0$	$H_0 : p = p_0$ $H_1 : p \neq p_0$
Test Statistic	$z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$		
Rejection Region			
Critical Value ($\alpha = 0.05$)	$z_\alpha = -1.645$	$z_\alpha = 1.645$	$z_{\alpha/2} = \pm 1.96$
Conclusion	Reject H_0 if $z < -1.645$ Otherwise, fail to reject H_0	Reject H_0 if $z > 1.645$ Otherwise, fail to reject H_0	Reject H_0 if $\lvert z \rvert > 1.96$ Otherwise, fail to reject H_0
Confidence Interval (2-tailed test)	$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\dfrac{p_0(1-p_0)}{n}}$ If H_0 is true, this CI gives the range of likely sample proportions $\hat{p}$. If $\hat{p}$ falls outside, reject H_0.		
Confidence Interval (Estimation)	$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ We are $100(1-\alpha)\%$ confident that the true population proportion p is within this interval.		

TWO-PROPORTION Z-TEST (Normal approximation)

	Left-tailed	**Right-tailed**	**Two-tailed**
Assumptions	• Observations are independent • Data should follow binomial distribution • Large sample: Typically, n_1 and n_2 should exceed 30 • Large sample: $n_i\hat{p}_i \geq 5$ and $n_i(1-\hat{p}_i) \geq 5$ for $i = 1, 2$ (some newer textbooks use ≥ 10)		
Hypotheses	$H_0 : p_1 - p_2 \geq 0$ $H_1 : p_1 - p_2 < 0$	$H_0 : p_1 - p_2 \leq 0$ $H_1 : p_1 - p_2 > 0$	$H_0 : p_1 - p_2 = 0$ $H_1 : p_1 - p_2 \neq 0$
Test Statistic	$z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{p_0(1-p_0)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}, \quad p_0 = \dfrac{y_1 + y_2}{n_1 + n_2}$		
Critical Values, Region & Conclusion	Same as one-proportion testing		
Confidence Interval (2-tailed test)	$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \cdot \sqrt{p_0(1-p_0)\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}$ If H_0 is true, this CI gives the likely range of sample differences $\hat{p}_1 - \hat{p}_2$. If the observed difference falls outside, reject H_0.		
Confidence Interval (Estimation)	$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ We are $100(1-\alpha)\%$ confident that the true difference in population proportions $p_1 - p_2$ is within this interval.		

Hypothesis Testing Summary

ONE-SAMPLE T-TEST

	Left-tailed	Right-tailed	Two-tailed
Assumptions	• Observations are independent • Population is approximately normal or $n \geq 30$ (Central Limit Theorem). • Population standard deviation σ is unknown. • No significant outliers and is a continuous data		
Hypotheses	$H_0 : \mu \geq \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu \leq \mu_0$ $H_1 : \mu > \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$
Test Statistic	$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$, where s is the sample standard deviation		
Critical Value	$t_{\alpha,df}$ with $df = n-1$	$t_{\alpha,df}$ with $df = n-1$	$\pm t_{\alpha/2,df}$ with $df = n-1$
Rejection Region	 $-t_{\alpha,df}$	 $t_{\alpha,df}$	 $-t_{\alpha/2,df}$ $t_{\alpha/2}$
Conclusion	Reject H_0 if $t < -t_\alpha$ Otherwise, fail to reject H_0	Reject H_0 if $t > t_\alpha$ Otherwise, fail to reject H_0	Reject H_0 if $\lvert t \rvert > t_{\alpha/2}$ Otherwise, fail to reject H_0
Confidence Interval	$\bar{x} \pm t_{\alpha/2,df} \cdot s/\sqrt{n}$ We are $100(1-\alpha)\%$ confident that the true mean μ lies within this interval. Also, if the null hypothesis $H_0 : \mu = \mu_0$ is true, then this interval shows the range of sample means $\bar{x}$ we would expect to observe with $100(1-\alpha)\%$ confidence.		

PAIRED T-TEST

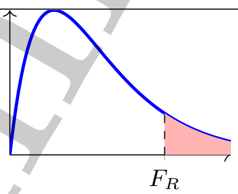
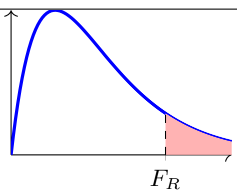
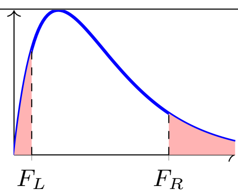
	Left-tailed	Right-tailed	Two-tailed
Assumptions	• Pairs are independent of each other. • Differences are approximately normally distributed (or $n \geq 30$ by CLT). • No significant outliers in the differences. • Data is continuous and collected in natural pairs (e.g., before/after, twins, A/B materials).		
Hypotheses	$H_0 : \mu_d \geq 0$ $H_1 : \mu_d < 0$	$H_0 : \mu_d \leq 0$ $H_1 : \mu_d > 0$	$H_0 : \mu_d = 0$ $H_1 : \mu_d \neq 0$
Test Statistic	$t = \dfrac{\bar{d}}{s_d/\sqrt{n}}$ where • $\bar{d}$ is the sample mean of the differences, • s_d is the sample standard deviation of the differences, and • n is the number of pairs.		
Critical Values, Region & Conclusion	Same as one-sample t-test		
Key Differences from One-Sample t-test	• Tests the mean of paired differences, not raw values. • The sample size n refers to the number of pairs (not total observations). • Observations within each pair are dependent, but the pairs themselves are independent.		

Hypothesis Testing Summary

TWO-SAMPLE T-TEST (Independent Samples)

	Left-tailed	Right-tailed	Two-tailed
Assumptions	• The two samples are independent of each other. • Data in each group is approximately normally distributed (or $n \geq 30$ by CLT). • No significant outliers. • For **Welch's t-test**: variances may be unequal. • For **pooled t-test**: equal variances (homogeneity of variances) with pooled standard deviation: $$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$		
Hypotheses	$H_0 : \mu_1 - \mu_2 \geq 0$ $H_1 : \mu_1 - \mu_2 < 0$	$H_0 : \mu_1 - \mu_2 \leq 0$ $H_1 : \mu_1 - \mu_2 > 0$	$H_0 : \mu_1 - \mu_2 = 0$ $H_1 : \mu_1 - \mu_2 \neq 0$
Test Statistic	**Equal variances (pooled t-test)** $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$	**Unequal variances (Welch's t-test)** $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$	
Degrees of freedom	**Equal variances (pooled t-test)** $df = n_1 + n_2 - 2$	**Unequal variances (Welch's t-test)** $df = \left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2 \Bigg/ \left[\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}\right]$	
t_α**, Region & Conclusion**	Same as one-sample t-test		
Confidence Interval	$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2,df} \cdot SE$ We are $100(1-\alpha)\%$ confident that the true difference in population means $\mu_1 - \mu_2$ lies within this interval.		

F-TEST: COMPARING TWO INDEPENDENT VARIANCES

	Left-tailed	Right-tailed	Two-tailed
Assumptions	• The two samples are independent. • Each population is normally distributed. • Each sample size is at least 30 for robustness.		
Hypotheses	$H_0 : \sigma_1^2 \geq \sigma_2^2$ $H_1 : \sigma_1^2 < \sigma_2^2$	$H_0 : \sigma_1^2 \leq \sigma_2^2$ $H_1 : \sigma_1^2 > \sigma_2^2$	$H_0 : \sigma_1^2 = \sigma_2^2$ $H_1 : \sigma_1^2 \neq \sigma_2^2$
Test Statistic	$F = \dfrac{s_2^2}{s_1^2}$ (reframe as right-tailed)	$F = \dfrac{s_1^2}{s_2^2}$	$F = \dfrac{\text{larger } s^2}{\text{smaller } s^2}$ (ensure $F \geq 1$)
Degrees of Freedom (df)	$v_1 = n_1 - 1, \quad v_2 = n_2 - 1$		
Critical Values	F_{α,v_1,v_2}	F_{α,v_1,v_2}	$F_{\alpha/2,v_1,v_2}$ and $\dfrac{1}{F_{\alpha/2,v_2,v_1}}$
Rejection Region	 $F > F_{\alpha,v_1,v_2}$	 $F > F_{\alpha,v_1,v_2}$	 $F < \dfrac{1}{F_{\alpha/2,v_2,v_1}}$ or $F > F_{\alpha/2,v_1,v_2}$
Conclusion	Reject H_0 if $F > F_R$ Otherwise, fail to reject H_0	Reject H_0 if $F > F_R$ Otherwise, fail to reject H_0	Reject H_0 if $F < F_L$ or $F > F_R$ Otherwise, fail to reject H_0

Hypothesis Testing Summary

CHI-SQUARED (χ^2) TEST

Component	**Goodness-of-Fit Test**	**Test of Independence**
Purpose	Test if observed frequencies match expected proportions in one categorical variable	Test if two categorical variables are independent in a contingency table
Assumptions	– Data are frequencies (counts), not percentages – Categories are mutually exclusive – Expected frequency in each category is at least 5	Same as for goodness-of-fit, applied to contingency table cells
Hypotheses	H_0: Proportions (in population) are equal to theoretical values proposed/stated H_1: Proportions (in population) are not equal to theoretical values proposed/stated	H_0: The variables are independent H_1: The variables are dependent or associated
Test Statistic	$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$	$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$, $E_{ij} = \frac{(\text{row total}) \cdot (\text{column total})}{\text{grand total}}$
Degrees of Freedom	$df = k - 1$ where k is the number of categories	$df = (r-1)(c-1)$, where r=rows, c=columns
Critical Value	$\chi^2_{\alpha,df}$ from chi-squared table	
Rejection Region	Reject H_0 if $\chi^2 > \chi^2_{\alpha,df}$	
Conclusion	If test statistic falls in rejection region, reject H_0. Otherwise, fail to reject H_0.	

Hypothesis Testing Decision Tree