SUPPLY CHAIN MODELLING AND OPTIMIZATION

TOPIC 2 SC NETWORK DESIGN – SINGLE-ECHELON SINGLE-COMMODITY

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TOPIC 2: NETWORK DESIGN – SESC

1. Introduction to Single-Echelon, Single-Commodity (SESC) Location Model
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RDCs = regional distribution centres
CDCs = central distribution centres

1 INTRODUCTION – NETWORK DESIGN

Network planning involves designing systems for moving goods from suppliers to demand points or selecting service facilities in the public sector. Key decisions include the **number, location, size**, and **equipment of facilities**, as well as whether to **divest, relocate, or downsize** them.

Classification of location problems

Location problems can be classified with respect to a number of criterias. For example,

Time horizon	Facility typology	Material flows	Dominant material flows
Single-period (make decision at the beginning)	Single-type (e.g. only RDCs)	Single-commodity (homogeneous items)	Single-echelon (inbound or outbound)
Multi-period (make decision in sequence)	Multi-type (both CDCs & RDCs)	Multi-commodity (several items)	Multiple-echelon (inbound and outbound)

1 INTRODUCTION – SESC

A **Single-Echelon, Single-Commodity (SESC) Location Model** is a logistics optimisation model used to select **optimal facility locations** (e.g., warehouses or distribution centers) in a **single-tiered distribution network**, handling only a **single type of product (commodity)**. This model simultaneously decides on facility locations and commodity distribution routes to **minimise overall logistics and operating costs**

Example. A petroleum company needs to locate fuel depots to serve gas stations across a region

Echelon:	Depot to station (one-tier system)
Commodity:	A single type of fuel (e.g., gasoline)
Goal:	Minimise operational costs and travel distance from depots to stations

2 SESC – ASSUMPTIONS

- Facilities are homogeneous (e.g., all regional warehouses).
- Only inbound or outbound flow is significant, not both (single-echelon).
- All flows are homogeneous (single-commodity).
- Transportation cost is linear or piecewise linear and increases at a decreasing rate as output or activity increases.
- Operating cost is piecewise linear and increases at a decreasing rate as output or activity increases (or constant).

3 STRUCTURE AND APPLICATIONS

The single-echelon, single-commodity network typically consists of **two node** types:

Facility Nodes (Potential Locations)

Candidate locations for warehouses or distribution centers.

Customer Nodes (Demand Points)

Locations requiring product delivery.

Some other applications

- Warehouse and distribution center location planning.
- Emergency response logistics (disaster relief centers).
- Public service facility location (fire stations, healthcare clinics).
- Retail outlet placement decisions.

4.1 CONSTANT COST MODEL FORMULATION

Decision Variables

- Facility Opening Decisions (z_j): Binary variables indicating if facility j is opened ($z_j = 1$) or not ($z_j = 0$).
- Commodity Flow Decisions (y_{jk}): Quantity of the commodity shipped from facility j to customer k.

Parameters

- Demand at customer k: D_k
- Capacity of facility j: K_j
- Fixed cost of opening facility j: f_j
- Transportation cost per unit from facility j to customer k: c_{jk}

4.1 CONSTANT COST MODEL FORMULATION

The objective function is to **minimise** total costs (facility opening + transportation)

$$\min \sum_{j\in V_1} \sum_{k\in V_2} c_{jk} y_{jk} + \sum_{j\in V_1} f_j z_j$$

Subject to constraints:

- **Demand satisfaction constraints** $$\sum_{j\in V_1} y_{jk} = D_k, \qquad \forall k \in V_2$$
- **Facility capacity constraints** $$\sum_{k\in V_2} y_{jk} \le K_j z_j, \qquad \forall j \in V_1$$
- **Binary capacity constraints** $$z_j \in \{0, 1\}, \qquad \forall j \in V_1$$
- **Non-negativity constraints** $$y_{jk} \ge 0, \qquad \forall j \in V_1, k \in V_2$$

4.1 CONSTANT COST – EXAMPLE

Example 1. A retailer must determine **optimal warehouse locations** to distribute a single commodity (e.g., bottled water) to three customer zones. Two warehouse locations are available (W1 and W2).

Table 1. Warehouse data

Warehouse	Capacity (units)	Fixed cost ($)
W1	300	1,000
W2	250	800

Table 2. Customer demands

Customer Zone	Demand (units)
Zone A	150
Zone B	120
Zone C	80

Table 3. Transportation cost from warehouse to customer zone ($/unit)

Warehouse	Zone A	Zone B	Zone C
W1	4	3	5
W2	2	4	3

4.1 CONSTANT COST – EXAMPLE

Sets and indices

- Set of warehouses: $V_1 = \{1, 2\}$, indexed by j
- Set of customer zones: $V_2 = \{A, B, C\}$, indexed by k

Decision Variables

- Facility opening decisions variables (z_j) indicating if warehouse j is opened $(z_j = 1)$ or not $(z_j = 0)$.
- Commodity Flow Decisions (y_{jk}): Quantity shipped from warehouse j to customer k.

Parameters

- Demand at customer zone k: D_k
- Capacity of warehouse j: K_j
- Fixed cost of opening warehouse j: f_j
- Transportation cost per unit from warehouse j to customer zone k: c_{jk}

4.1 CONSTANT COST – EXAMPLE

Objective function

$$\min Z = \min \sum_{j \in V_1} \sum_{k \in V_2} c_{jk} y_{jk} + \sum_{j \in V_1} f_j z_j$$

Explicitly, for this scenario:

$$\min Z = (4y_{1A} + 3y_{1B} + 5y_{1C}) + (2y_{2A} + 4y_{2B} + 3y_{2C}) + 1000z_1 + 800z_2$$

Constraints

- **Demand satisfaction constraints** (each customer zone's demand must be satisfied)

$$\sum_{j \in V_1} y_{jk} = y_{1k} + y_{2k} = D_k \quad \begin{cases} y_{1A} + y_{2A} = 150 \\ y_{1B} + y_{2B} = 120 \\ y_{1C} + y_{2C} = 80 \end{cases}$$

- **Warehouse capacity constraints**

$$\sum_{k \in V_2} y_{jk} \le K_j z_j \quad \begin{cases} y_{1A} + y_{1B} + y_{1C} \le 300z_1 \\ y_{2A} + y_{2B} + y_{2C} \le 250z_2 \end{cases}$$

- **Binary capacity constraints** $z_j \in \{0, 1\}, \quad \forall j \in V_1$
- **Non-negativity constraints** $y_{jk} \ge 0, \quad \forall j \in V_1, k \in V_2$

Note. SESC model is commonly solved using **Mixed-Integer Linear Programming** (**MILP**), because binary decision variables are not suitable for Lagrangian method. Also, manual calculation will become infeasible as the number of variables and the complexity of the constraints and cost structure increase.

4.1 CONSTANT COST – EXAMPLE

First, let's check all the possible configuration of (z_1, z_2).

There are 3 possible combinations (excluding $z_1 = z_2 = 0$): $(0, 1),\ (1, 0),\ (1, 1)$

For all possible combinations, the total capacity need to at least meet the total demand.

$$\text{Total Demand} = \sum_{k \in V_2} D_k = 150 + 120 + 80 = 350$$

$$\text{Total Capacity} = \sum_{j \in V_1} K_j z_j = 300z_1 + 250z_2$$

Case 1: Warehouse 2 Open
$(\boldsymbol{z_1 = 0, z_2 = 1})$

$\text{Total Capacity} = 250 < 350$

Infeasible (skip this case)

Case 2: Warehouse 1 Open
$(\boldsymbol{z_1 = 1, z_2 = 0})$

$\text{Total Capacity} = 300 < 350$

Infeasible (skip this case)

Case 3: Both Warehouses Open
$(\boldsymbol{z_1 = 1, z_2 = 1})$

$\text{Total Capacity} = 550 > 350$

Feasible (continue)

If the problem itself is fairly simple (≤ 3 facilities and ≤ 3 customers), we can evaluate the objective function for each possible combinations and pick the lowest Z. Otherwise, computer programming is required.

4.1 CONSTANT COST – EXAMPLE

Consider the case where **both warehouses open** ($z_1 = 1, z_2 = 1$), let's assign the lowest-cost routes based on the information given in Table 2 and Table 3.

Table 3. Transportation cost from warehouse to customer zone ($/unit)

Warehouse	Zone A	Zone B	Zone C
W1	4	3	5
W2	2	4	3

Table 2. Customer demands

Customer Zone	Demand (units)
Zone A	150
Zone B	120
Zone C	80

Zone A: The cheapest route is from W2; therefore, assign All of A's to W2 → $y_{2A} = 150$

Zone B: The cheapest route is from W1; therefore, assign All of B's to W1 → $y_{1B} = 120$

Zone C: The cheapest route is from W2; therefore, assign All of C's to W2 → $y_{2C} = 80$

Verify the capacity used: W1 = 120 < 300 ✓ and W2 = 150 + 80 = 230 < 250 ✓

Objective Function: $Z = 2(150) + 3(80) + 3(120) + 1000 + 800 = 2700$

4.1 CONSTANT COST – EXAMPLE

Alternatively, we can use Excel Solver tool, which allows us to alter decision variables in a spreadsheet in order to achieve a desired objective function.

How to begin?

1. Setup costs, capacities and demand input table
2. Setup decision variables table for y_{jk} and z_j
3. Setup constraint tables using Excel formula
   i. Capacity constraints can be written as

   $$K_j z_j - \sum_k y_{jk} \geq 0$$

   ii. Demand constraints can be written

   $$D_k - \sum_j y_{jk} = 0$$
4. Setup objective function
5. Initialise decision variables and cost as zero.

Input - Costs, Capacities, Demand					
	Zone A	Zone B	Zone C	Cost	Capacity
W1	4	3	5	1000	300
W2	2	4	3	800	250
Demand	150	120	80		

Decision Variables y_jk				
	Zone A	Zone B	Zone C	z_j
W1	0	0	0	0
W2	0	0	0	0

Constraints

Warehouse	Capacity
W1	0
W2	0

Unmet Demand	Zone A	Zone B	Zone C
	150	120	80

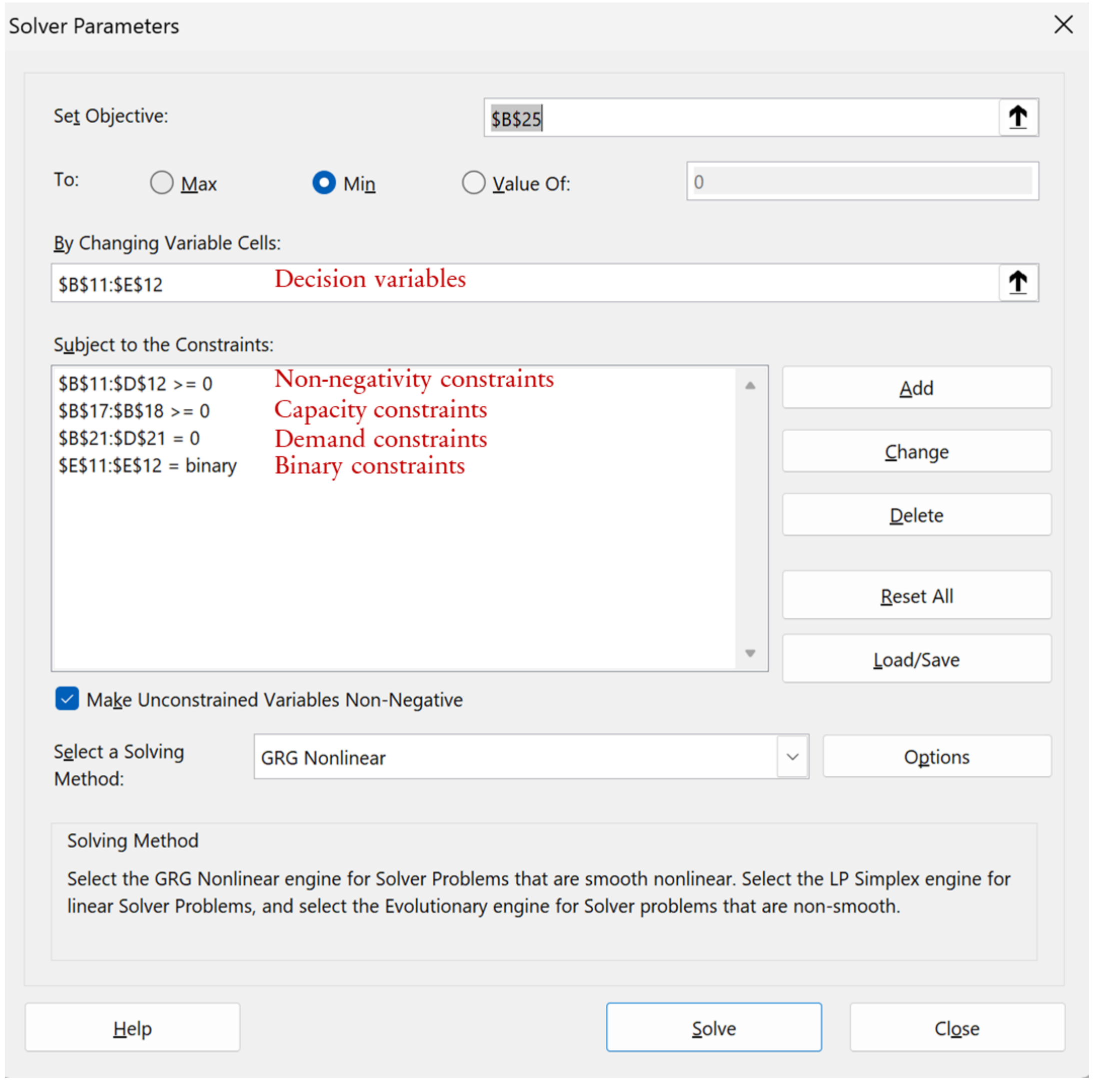
Objective function	
Cost =	0

4.1 CONSTANT COST – EXAMPLE

How to use Excel Solver tool in Microsoft Excel?

1. File → Options → Add-ins
   → Manage Excel Add-ins → Go
   → Tick on **Solver Add-in** → OK
2. From **Data** tab, check under **Analyze** → **Solver**
3. On **Solver Parameters** window,
   → Set Objective: select a cell and set a goal
   → Add constraints
   → Solve

Check your Excel Solver solution with our solution.



4.1 CONSTANT COST – EXAMPLE

Input - Costs, Capacities, Demand					
	Zone A	Zone B	Zone C	Cost	Capacity
W1	4	3	5	1000	300
W2	2	4	3	800	250
Demand	150	120	80		

Decision Variables y_jk				
	Zone A	Zone B	Zone C	z_j
W1	0	120	0	1
W2	150	0	80	1

Constraints	
Warehouse	Capacity
W1	180
W2	20

Unmet Demand	Zone A	Zone B	Zone C
	0	0	0

Objective function	
Cost =	**2700**

Interpret the results

- Both warehouses should be opened.
- W1 should ship 120 units to Zone B.
- W2 should ship 150 units to Zone A and 80 units to Zone C.

- There are 180 units of capacity left in W1.
- There are 20 units of capacity left in W2.

- The minimum total cost is $2,700

4.2 SINGLE-SOURCING CONDITION

The single-sourcing condition in a SESC facility location model refers to a constraint that enforces each customer (or demand point) to be served by exactly one facility (e.g., warehouse, depot, plant) — not split across multiple facilities.

In standard SESC models, the shipment variable y_{jk} represents the amount of product shipped from facility j to customer k. Under single-sourcing, each customer k must receive its entire demand from only one facility.

Mathematically

Let $x_{jk} \in \{0,1\}$: binary assignment – if facility j serves customer k, 0 otherwise.

1. Demand assigned to one source only: $\sum_{j} x_{jk} = 1, \quad \forall k$

2. Flow consistency (linking constraint): $y_{jk} = D_k \cdot x_{jk}, \qquad \forall j, k$

Shipments match full demand only if facility j is assigned to serve customer k.

4.2 SINGLE-SOURCING CONDITION – EXAMPLE

Example 2. An emergency response organisation selects from four potential distribution facilities (F1, F2, F3, F4) to supply drinking water to six affected areas (A, B, C, D, E, F) following a natural disaster. Each area's demand must be satisfied entirely from exactly one distribution facility.

Table 1. Facility information

Facility	Capacity (units)	Fixed Cost ($)
F1	400	2000
F2	350	1800
F3	450	2200
F4	500	2500

Table 2. Affected area demand

Affected Area	A	B	C	D	E	F
Demand (units)	150	100	180	150	120	100

Table 3. Transportation cost ($/unit)

Facility	A	B	C	D	E	F
F1	3	4	5	4	3	4
F2	4	3	2	5	4	3
F3	3	2	5	4	3	5
F4	5	3	4	2	3	4

4.2 SINGLE-SOURCING CONDITION – EXAMPLE

Sets and indices

- Set of facilities: $J = \{1, 2, 3, 4\}$, indexed by j
- Set of customer zones: $K = \{A, B, C, D, E, F\}$, indexed by k

Decision Variables

- Facility opening decisions variables (z_j): $z_j = \begin{cases} 1, & \text{if facility } j \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$
- Assignment decision (single-sourcing) (x_{jk}): $x_{jk} = \begin{cases} 1, & \text{if area } k \text{ is supplied by facility } j \\ 0, & \text{otherwise} \end{cases}$
- Commodity Flow Decisions (y_{jk}): Quantity shipped from warehouse j to customer k.

Parameters

- Demand at customer zone k: D_k
- Transportation cost per unit from facility j to area k: c_{jk}
- Capacity of facility j: K_j
- Fixed cost of opening warehouse j: f_j

4.2 SINGLE-SOURCING CONDITION – EXAMPLE

Objective function

$$\min Z = \min \sum_{j \in J} \sum_{k \in K} c_{jk} y_{jk} + \sum_{j \in J} f_j z_j$$

Constraints

- **Demand satisfaction constraints** (single-sourcing)

$$\sum_{j \in J} x_{jk} = 1, \qquad \forall k \in K$$

$$y_{jk} = D_k \cdot x_{jk}, \qquad \forall j \in J, k \in K$$

- **Facility capacity constraints**

$$\sum_{k \in K} y_{jk} \leq K_j z_j\,, \qquad \forall j \in J$$

- **Facility opening constraints**

$$x_{jk} \leq z_j, \qquad \forall j \in J, k \in K$$

- **Binary capacity constraints**

$$z_j, x_{jk} \in \{0, 1\}, \qquad \forall j \in J, k \in K$$

- **Non-negativity constraints**

$$y_{jk} \geq 0, \qquad \forall j \in J, k \in K$$

Input - Costs, Capacities, Demand

	A	B	C	D	E	F	Cost	Capacity
F1	3	4	5	4	3	4	2000	400
F2	4	3	2	5	4	3	1800	350
F3	3	2	5	4	3	5	2200	450
F4	5	3	4	2	3	4	2500	500
Demand	150	100	180	150	120	100		

Decision Variables

y_jk	A	B	C	D	E	F	z_j
F1	0	0	0	0	0	0	0
F2	150	0	180	0	0	0	1
F3	0	0	0	0	0	0	0
F4	0	100	0	150	120	100	1

x_jk	A	B	C	D	E	F
F1	0	0	0	0	0	0
F2	1	0	1	0	0	0
F3	0	0	0	0	0	0
F4	0	1	0	1	1	1

Constraints

Facility	Capacity	Assignment					
F1	0	0	0	0	0	0	0
F2	20	0	1	0	1	1	1
F3	0	0	0	0	0	0	0
F4	30	1	0	1	0	0	0

Demand	A	B	C	D	E	F
F1	0	0	0	0	0	0
F2	0	0	0	0	0	0
F3	0	0	0	0	0	0
F4	0	0	0	0	0	0
sum x_jk	1	1	1	1	1	1

Objective function

Cost =	6620

Interpret the results

- F2 and F4 should be opened.
- F2 should ship to Area A and C.
- F4 should ship to Area B, D, E, and F

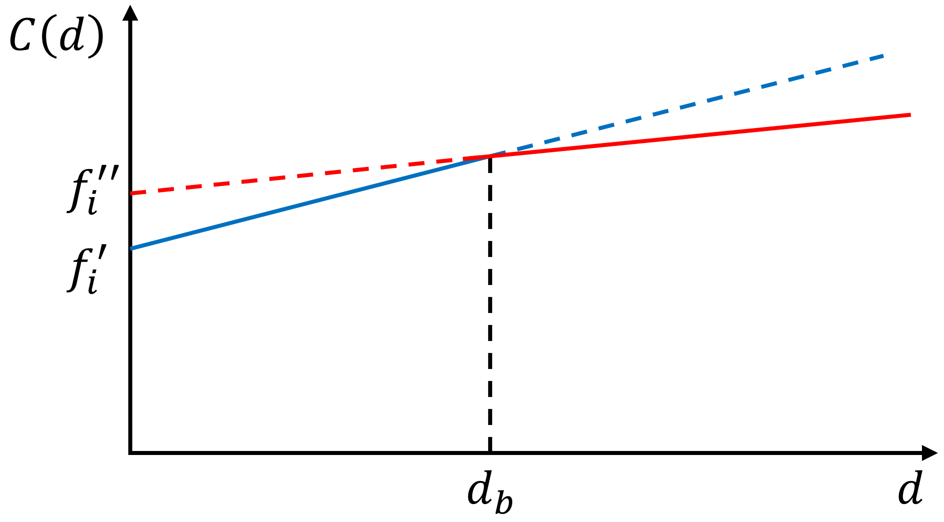
- There are 20 units of capacity left in F2.
- There are 30 units of capacity left in F4.

- The minimum total cost is $6,600

5.1 PIECEWISE COST MODEL

The operating cost of a potential facility is usually a **piecewise linear function**. In the simplest case, there are only two piecewise lines. The operating costs of potential facility i, $C(d)$, for a particular demand d are characterised by fixed costs (f_i' and f_i'') and marginal costs (g_i' and g_i''). Then,

$$C(d) = \begin{cases} f_i' + g_i' \cdot d & \text{if } 0 < d \leq d_b, \\ f_i'' + g_i'' \cdot d & \text{if } d > d_b, \\ 0 & \text{if } d = 0. \end{cases}$$

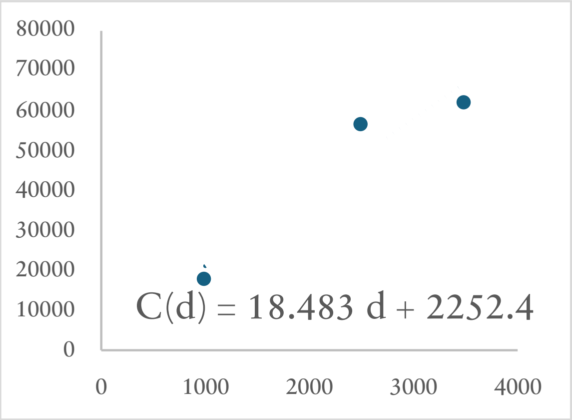


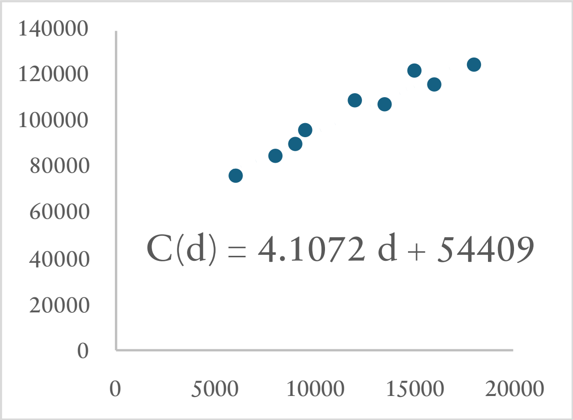
5.1 PIECEWISE COST MODEL – EXAMPLE

Example 3. Historical demand and costs data of a firm are given as follows:

d	1000	2500	3500	6000	8000	9000	9500	12000	13500	15000	16000	18000
C	17579	56350	62208	76403	85491	90237	96251	109429	10755	122432	116816	124736

Use the above data to model the relationship between demand and cost as a piecewise linear function with a known breakpoint at $d = d_b = 3500$. Explain the results and formulate the optimisation model.





$$C(d) = \begin{cases} f_i' + g_i' \cdot d & \text{if } 0 < d \leq d_b, \\ f_i'' + g_i'' \cdot d & \text{if } d > d_b, \\ 0 & \text{if } d = 0. \end{cases} \quad \Rightarrow \quad C(d) = \begin{cases} 2252.4 + 18.483d & \text{if } d \leq 3500, \\ 54409.4 + 4.107d & \text{if } d > 3500, \\ 0 & \text{if } d = 0. \end{cases}$$

5.1 PIECEWISE COST MODEL – EXAMPLE

A company is considering network design decisions over **two planning periods** and deciding between **two warehouse locations** (A and B) to serve **three market regions** (X, Y, and Z). Demand is uncertain and may vary between periods. The company must determine which warehouse(s) to open and how to allocate demand in each period to minimise total costs, including both fixed and transportation costs.

Table 1. Capacity, demand and transportation cost

Warehouse	**Market X**	**Market Y**	**Market Z**	**Fixed Cost ($)**	**Capacity (tonnes)**
A	200	300	100	50,000	60
B	250	150	200	40,000	50
Period 1 Demand	30	40	20		

Potential Demand Changes (Period 2)

- Scenario 1: Market X increases by 10%, Y decreases by 20%, Z remains constant.
- Scenario 2: Market X decreases by 15%, Y increases by 25%, Z increases by 10%.

5.1 PIECEWISE COST MODEL – EXAMPLE

Objective Function (with Piecewise Cost)

Minimise total cost

$$\min Z = \min \sum_{j \in J} \sum_{k \in K} C_{jk} + \sum_{j \in J} f_j z_j$$

where

- f_j: fixed cost for opening warehouse j
- $z_j \in \{0, 1\}$: binary variable; 1 if warehouse j is open
- C_{jk}: cost of serving market k from warehouse j, defined as:

$$C_{jk} = \begin{cases} f'_{jk} + g'_{jk} \cdot y_{jk}, & \text{if } y_{jk} \leq d_b, \\ f''_{jk} + g''_{jk} \cdot y_{jk}, & \text{if } y_{jk} > d_b, \end{cases}$$

- $w_{jk} \in \{0, 1\}$: binary variable indicating which piece of the cost function is active:

$$w_{jk} = 0 \rightarrow \text{lower segment is active (i.e. } y_{jk} \leq d_b)$$
$$w_{jk} = 1 \rightarrow \text{upper segment is active (i.e. } y_{jk} > d_b)$$

5.1 PIECEWISE COST MODEL – EXAMPLE

Constraints

1. Demand satisfaction (Single sourcing) $\sum_{j \in J} x_{jk} = 1, \quad \forall k \in K, \quad y_{jk} = D_k \cdot x_{jk}, \quad \forall j \in J, k \in K$

2. Facility capacity constraints $\sum_{k \in K} y_{jk} \leq K_j z_j, \quad \forall j \in J$

   Facility opening constraints $x_{jk} \leq z_j, \quad \forall j \in J, k \in K$

3. Piecewise cost approximation (Big-M Form)

$$C_{jk} \geq f'_{jk} + g'_{jk} \cdot y_{jk} - M w_{jk}$$ Lower segment constraint

$$C_{jk} \geq f''_{jk} + g''_{jk} \cdot y_{jk} - M(1 - w_{jk})$$ Upper segment constraint

$$y_{jk} \leq d_b + M w_{jk}$$ Breakpoint enforcement from below

$$y_{jk} \geq d_b + \epsilon - M(1 - w_{jk})$$ Breakpoint enforcement from above

4. Binary capacity constraints $z_j, x_{jk}, w_{jk} \in \{0, 1\}, \quad \forall j \in J, k \in K$

5. Non-negativity constraints $y_{jk} \geq 0, \quad \forall j \in J, k \in K$

Big M is a large positive constant used in MILP to enable or disable constraints depending on the value of a binary variable (like 0 or 1).

5.1 PIECEWISE COST MODEL – EXAMPLE

Piecewise cost approximation (Big-M Form) explain

Lower segment constraint $C_{jk} \geq f'_{jk} + g'_{jk} \cdot y_{jk} - Mw_{jk}$

- When $w_{jk} = 0$, this become $C_{jk} \geq f'_{jk} + g'_{jk} \cdot y_{jk}$ → **active lower piece**
- When $w_{jk} = 1$, subtracting big M allows this constraint to be trivially satisfied → **inactive**

Breakpoint enforcement from above $y_{jk} \geq d_b + \epsilon - M(1 - w_{jk})$

- When $w_{jk} = 1$, this become $y_{jk} \geq d_b + \epsilon$, where ϵ is a tiny number to ensure separation
- When $w_{jk} = 0$, RHS becomes large negative → always holds.

Exercise: interpret the upper segment constraint and breakpoint enforcement from below.