SUPPLY CHAIN MODELLING AND OPTIMIZATION

TOPIC 3 SC NETWORK DESIGN – TWO-ECHELON MULTICOMMODITY LOCATION MODEL (TEMC)

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TOPIC 3: NETWORK DESIGN – TEMC

1. Introduction to Two-Echelon, Multi-Commodity (TEMC) Location Model
2. Commodity types
3. Structure
4. Objectives, decision variables, and constraints
5. TEMC in action
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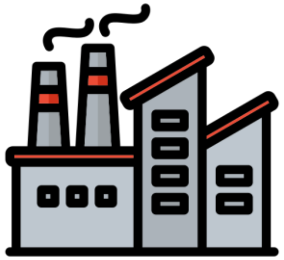
6. Tutorial Exercises

1 INTRODUCTION – TEMC

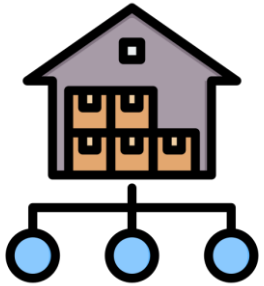
A **Two-Echelon Multicommodity Location Model (TEMC)** is an optimisation framework used in logistics and supply chain management for decisions involving facility location and routing of **multiple commodities** across a **two-tier distribution system**. It involves optimal selection of intermediate facilities (e.g., distribution centers) and routing of commodities from origins to destinations.

Example. Global supply chain for a supermarket chain (e.g., Walmart or Carrefour)



Echelon 1
(Upper level)

CDCs
Bulk shipment



Echelon 2
(Lower level)

RDCs
Smaller, store-ready shipment

Customer Nodes
(Demand Points)

Retail Stores
Multicommodity

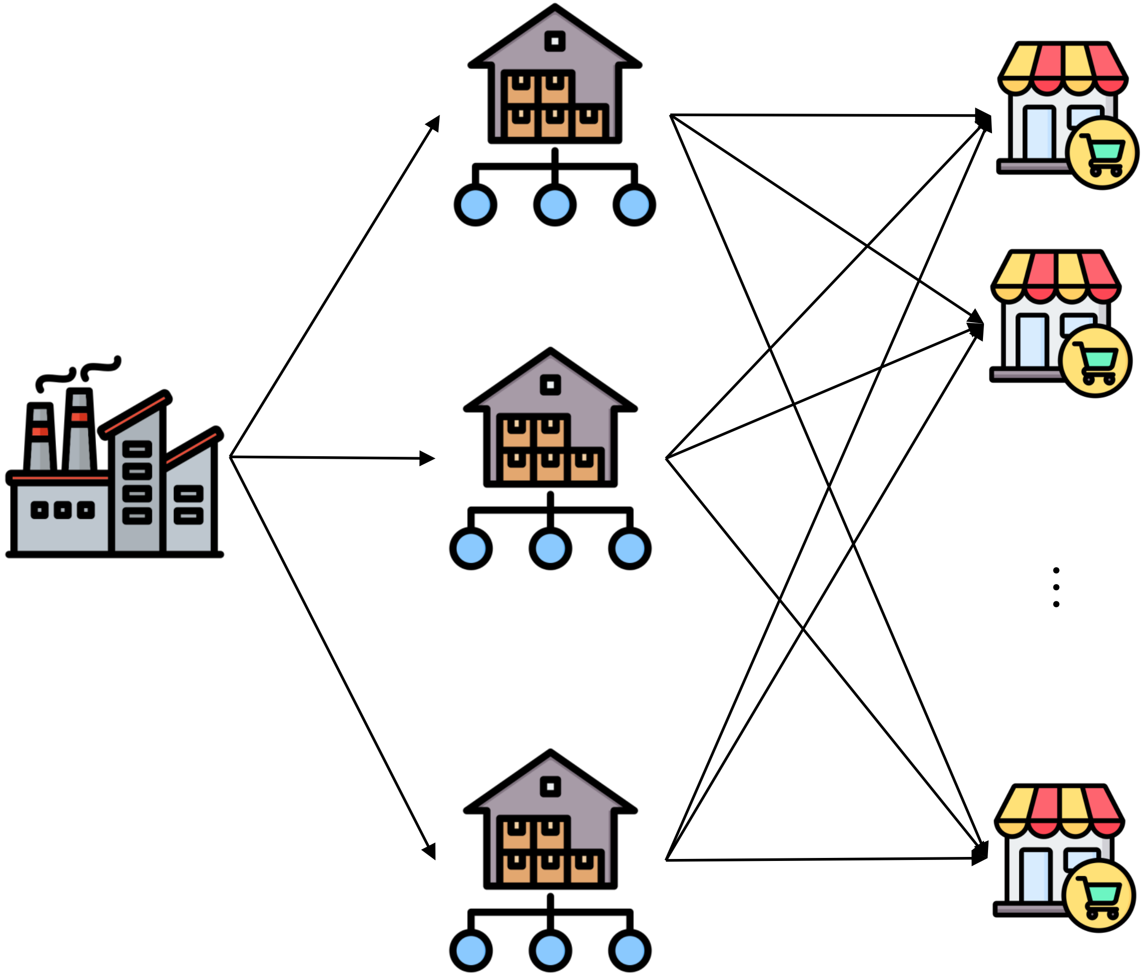
Commodities: Fresh produce (perishable), packaged food (non-perishable), personal care, electronics (low volume, high value)

Multicommodity flows: Manages multiple products with distinct characteristics.

2 COMMODITY TYPES

Type	Description	Examples
Perishable goods	Require refrigeration/fast delivery	Fresh fruits and vegetables, meat, daily products (milk, cheese)
Packaged consumer goods (non-perishable)	Standard shelf-stable products	Canned food, snacks, beverages, cleaning supplies, detergents
Health and personal care	Often require clean handling, sometimes regulated	Medicines and pharmaceuticals, cosmetics
Electronics and Appliances	Sensitive, often high-value	Smart phones, laptops, home appliances, batteries and chargers
Industrial goods and raw materials	Used in manufacturing or construction	Steel, copper, aluminum, machinery parts, building materials, chemicals
Apparel and textiles	Non-perishable, high volume	Clothing, shoes, beddings, fabrics
Hazardous materials	Special safety and transport regulations apply	Flammable liquids, gasses under pressure, toxic substances, explosives
Automotive products	Bulk or spare parts	Tires, engine parts, lubricants, windshields

3 STRUCTURE



Decision to make in the model

1. Where to locate:
   - CDCs (tier 1)
   - RDCs (tier 2)
2. Which facility serves whom:
   - Which RDCs are supplied by which CDCs
   - Which RDCs serve which retail stores
3. How much each commodity flows:
   - From CDC → RDC
   - From RDC → Stores

Throughtput = the amount of material or items passing through a system or process

4 OBJECTIVES, DECISION VARIABLES & CONSTRAINTS

Objectives

- Minimising fixed facility opening costs
- Minimising total transportation costs for each commodity and path
- Minimising possible storage/handling costs
- Ensuring service-level compliance

Decision variables

- **Facility Location Variables:** Indicate opening of DCs.
- **Commodity Allocation Variables:** Routing of commodities.

Constraints

- **Capacity constraints:** Maximum throughput at each facility.
- **Demand fulfillment:** Customer demands must be satisfied.
- **Flow conservation:** Balance of commodity flows at intermediate nodes.

5.1 TEMC IN ACTION – EXAMPLE 1

- **Echelon 1 (Suppliers → Distribution Centers):**
  - Suppliers (Factories): S1, S2
  - Potential Distribution Centers: DC1, DC2
- **Echelon 2 (Distribution Centers → Customers):**
  - Customers: C1, C2, C3
- **Commodities:** Products: P1,P2

Table 1: Supplier Capacities and Supplier to DC transportation cost

Supplier	Product	Capacity (units)	Cost to DC1 ($/unit)	Cost to DC2 ($/unit)
S1	P1	100	3	4
S1	P2	150	3	4
S2	P1	200	2	5
S2	P2	100	2	5

Table 3: Customer Demands (units)

Customer	Demand P1	Demand P2
C1	80	50
C2	120	100
C3	60	70

Table 2: DC capacity and DC to Customer transportation cost

DC	Capacity (units)	Cost to C1 ($/unit)	Cost to C2 ($/unit)	Cost to C3 ($/unit)
DC1	300	4	3	6
DC2	250	5	4	2

Table 4: Fixed and variable costs of DCs

DC	Fixed Cost $	Handling Cost/unit $
DC1	500	1
DC2	400	1.5

5.1 TEMC IN ACTION – EXAMPLE 1

i. Which distribution centers (DC1, DC2, or both) should be operational to minimise total cost?
ii. How much of each product (P1, P2) should be shipped from each supplier (S1, S2) to each selected DC?
iii. How much of each product should be shipped from the selected DCs to each customer (C1, C2, C3)?
iv. What is the optimal allocation strategy to minimise total costs (including transportation, fixed, and handling costs)?

1. Set model variables

$$
\begin{aligned}
V_1 &= \{\mathrm{S1}, \mathrm{S2}\} && \text{(Suppliers)} \\
V_2 &= \{\mathrm{DC1}, \mathrm{DC2}\} && \text{(Distribution Centers)} \\
V_3 &= \{\mathrm{C1}, \mathrm{C2}, \mathrm{C3}\} && \text{(Customers)} \\
H &= \{\mathrm{P1}, \mathrm{P2}\} && \text{(Products)}
\end{aligned}
$$

2. Model parameters

- Supplier Capacities: p_i^h
- DC Capacities: K_j
- Customer Demand: D_k^h
- Transportation Costs: c_{ijk}^h
- Fixed Costs of DCs: f_j
- Variable Handling Costs of DCs: g_j

5.1 TEMC IN ACTION – EXAMPLE 1

3. Decision Variables

- $x^h_{ijk} \geq 0$: Flow quantity of product h from supplier i via DC j to customer k.
- $y_{jk} \in \{0,1\}$: Binary indicator if DC j serves customer k.
- $z_j \in \{0,1\}$: Binary indicator if DC j is opened.

4. Objective Function

$$\text{Minimize} \quad \sum_{i\in V_1}\sum_{j\in V_2}\sum_{k\in V_3}\sum_{h\in H} c^h_{ijk}x^h_{ijk} + \sum_{j\in V_2}\left(f_j z_j + g_j \sum_{k\in V_3}\sum_{h\in H} D^h_k y_{jk}\right)$$

Transportation cost | **Facility opening cost** | **Variable handling cost**

- **Transportation cost:** (Supplier → DC + DC → Customer)

  E.g. Transportation cost from S1 → DC1 → C1 is given by $\sum_{h\in\{P1,P2\}} [c^h_{S1,DC1} + c^h_{DC1,C1}] \cdot x^h_{S1,DC1,C1}$

$$\left[c^{P1}_{S1,DC1} + c^{P1}_{DC1,C1}\right] \cdot x^{P1}_{S1,DC1,C1} + \left[c^{P2}_{S1,DC1} + c^{P2}_{DC1,C1}\right] \cdot x^{P2}_{S1,DC1,C1}$$

How many transportation terms do we have in total?

5.1 TEMC IN ACTION – EXAMPLE 1

4. **Objective Function (Continue)**

- **Variable handling cost** $\sum_{j \in V_2} g_j \sum_{k \in V_3} \sum_{h \in H} D_k^h y_{jk}$ cost of processing all demanded units at each DC

5. **Constraints**

(i) **Supplier capacity constraints**

For each supplier i and product h, the total quantity shipped (via any DC j to any customer k) must not exceed the supplier's capacity for that product.

$$\sum_{j \in V_2} \sum_{k \in V_3} x_{ijk}^h \leq p_i^h, \quad \forall i \in V_1, h \in H$$

E.g. for Supplier S1, Product P1:

$$x_{S1,DC1,C1}^{P1} + x_{S1,DC1,C2}^{P1} + x_{S1,DC1,C3}^{P1} + x_{S1,DC2,C1}^{P1} + x_{S1,DC2,C2}^{P1} + x_{S1,DC2,C3}^{P1} \leq 100$$

How many supplier capacity constraints do we have?

5.1 TEMC IN ACTION – EXAMPLE 1

5. **Constraints (Continue)**

   (ii) **Demand satisfaction constraints (under single-sourcing)**

$$\sum_{i \in V_1} x^h_{ijk} = D^h_k y_{jk}, \quad \forall j \in V_2, k \in V_3, h \in H$$

Each customer *k* must receive their full demand for product *h* only from the DC *j* that serves them

$$\sum_{j \in V_2} y_{jk} = 1, \quad \forall k \in V_3$$

= 1 if DC *j* serves customer *k*

E.g. for Customer C1 and Product P1:

$$x^{P1}_{S1,DC1,C1} + x^{P1}_{S2,DC1,C1} = 80 y_{DC1,C1}$$

If DC1 serves C1, it must deliver all 80 units of P1 via flows from S1 and/or S2

$$y_{DC1,C1} + y_{DC2,C1} = 1$$

Ensures that each customer is assigned to exactly one DC (i.e., single-sourcing)

5.1 TEMC IN ACTION – EXAMPLE 1

5. **Constraints** (**Continue**)

(iii) **DC capacity constraints**

$$\sum_{k \in V_3} \sum_{h \in H} D_k^h y_{jk} \leq K_j z_j, \quad \forall j \in V_2$$

For each distribution center j, the total demand it is responsible for (across all customers and all products) must not exceed its capacity K_j — and only if it's open ($z_j = 1$).

E.g. DC1 capacity constraint

$$(80 + 50)y_{DC1,C1} + (120 + 100)y_{DC1,C2} + (60 + 70)y_{DC1,C3} \leq 250z_{DC1}$$

(iv) **DC opening constraints**

$$y_{jk} \leq z_j, \quad \forall j \in V_2, k \in V_3$$

Links customer assignment to the DC opening decision

(v) **Number of DCs constraint**

$$\sum_{j \in V_2} z_j = p$$

Ensure that exactly p DCs are opened, depending on the objectives.

(vi) **Variable bounds**

$$x_{ijk}^h \geq 0, \quad y_{jk} \in \{0,1\}, \quad z_j \in \{0,1\}$$

5.1 TEMC IN ACTION – EXAMPLE 1

Optimal solution

- Open both DCs
- DC1 serves customer C1 & C3
- DC2 serves customer C2

i	j	k	h	x_{ijk}^h	$c_{ijk}^h = c_{ij}^h + c_{jk}^h$		g_j		
SUP	**DC**	**CUS**	**Product**	**Flow Units**	**S → DC ($/unit)**	**DC → C ($/unit)**	**Handling cost ($/unit)**	**Total (w/o fixed cost)**	
S1	DC1	C1	P1	40	3	4	1	320	(3+4+1)(40)
S1	DC1	C1	P2	50	3	4	1	400	(3+4+1)(50)
S1	DC1	C3	P1	60	3	6	1	600	(3+6+1)(60)
S2	DC1	C1	P1	40	2	4	1	280	
S2	DC1	C3	P2	70	2	6	1	630	
S1	DC2	C2	P2	100	4	4	1.5	950	
S2	DC2	C2	P1	120	5	4	1.5	1260	

$$\textbf{Minimum total costs} = 4440 + \underbrace{500(1)}_{f_{DC1}z_{DC1}} + \underbrace{400(1)}_{f_{DC2}z_{DC2}} = \$5{,}340$$

Note. We can also calculate each term separately like in the old example.

5.2 TEMC IN ACTION – EXAMPLE 2

An electronics company plans to optimize the distribution of two products, Smartphones (P1) and Laptops (P2), from two factories through two potential distribution centers (DCs), to three customer regions.

Echelon Structure

- **Echelon 1 (Suppliers → DCs)**
  - Suppliers: Factory A (FA), Factory B (FB)
  - Distribution Centres: DC East (DCE), DC West (DCW)
- **Echelon 2 (DCs → Customers)**
  - Customer Regions: Region Central (RC), Region South (RS), Region North (RN)

Table 1: Supplier Capacities (units)

Supplier	Smartphones (P1)	Laptops (P2)
FA	500	300
FB	400	400

Table 2: DC Capacities, Fixed Costs, and Handling Costs

DC	Capacity	Fixed Cost ($)	Handling Cost ($/unit)
DCE	800	1000	2
DCW	900	1200	1.5

Table 3: Customer Demands (units)

Customer	Smartphones (P1)	Laptops (P2)
RC	200	150
RS	300	250
RN	200	150

Table 4 & 5: Transportation costs ($/unit)

Supplier	DCE	DCW
FA	5	7
FB	4	6

DC	RC	RS	RN
DCE	4	7	3
DCW	5	2	4

5.2 TEMC IN ACTION – EXAMPLE 2

1. Set and Indices

- I: Set of suppliers (production plants), indexed by i
- J: Set of candidate distribution centers (DCs), indexed by j
- K: Set of customers (demand regions), indexed by k
- H: Set of products (commodities), indexed by h

2. Decision Variables

- x_{ij}^h: Quantity of product h shipped from supplier i to DC j
- y_{jk}^h: Quantity of product h shipped from DC j to customer k
- z_j: Binary variable indicating if DC j is opened (1 if opened, 0 otherwise)

3. Parameters

- P_i^h: Production capacity of supplier i for product h
- K_j: Maximum capacity of DC j
- D_k^h: Demand of customer k for product h
- C_{ij}: Transportation cost per unit from supplier i to DC j
- C_{jk}: Transportation cost per unit from DC j to customer k
- f_j: Fixed cost of opening DC j
- g_j: Handling cost per unit at DC j

5.2 TEMC IN ACTION – EXAMPLE 2

4. Objective Function

$$\min \sum_{i\in I}\sum_{j\in J}\sum_{h\in H} C_{ij}x_{ij}^h + \sum_{j\in J}\sum_{k\in K}\sum_{h\in H} C_{jk}y_{jk}^h + \sum_{j\in J}\left(f_j z_j + g_j \sum_{k\in K}\sum_{h\in H} y_{jk}^h\right)$$

Supplier → DC transportation cost

DC → Customer transportation cost

DC fixed + handling cost

Mathematically equivalent to Example 1's transportation costs if we define $C_{ijk}^h = C_{ij}^h + C_{jk}^h$

y_{jk}^h is a flow variable not binary; thus, we don't need to multiply D_k^h

5. Constraints

- **Supplier Capacity Constraints:** Total shipments from a supplier must not exceed its capacity:

$$\sum_{j\in J} x_{ij}^h \leq P_i^h, \quad \forall i \in I, h \in H$$

5.2 TEMC IN ACTION – EXAMPLE 2

5. Constraints (continue)

- **DC Capacity Constraints:** DC shipments must not exceed the DC's capacity if opened:

$$\sum_{k \in K} \sum_{h \in H} y_{jk}^h \leq K_j z_j, \quad \forall j \in J$$

- **Demand Fulfillment Constraints:** Demand for each product at each customer must be fully met:

$$\sum_{j \in J} y_{jk}^h = D_k^h, \quad \forall k \in K, h \in H$$

- **Flow Conservation at DCs:** The quantity entering a DC must equal quantity leaving:

$$\sum_{i \in I} x_{ij}^h = \sum_{k \in K} y_{jk}^h, \quad \forall j \in J, h \in H$$

- **Facility Opening Constraint (Binary constraint):** Ensure that DC opening decisions remain binary:

$$z_j \in \{0, 1\}, \quad \forall j \in J$$

- **Non-negativity Constraints**

$$x_{ij}^h, y_{jk}^h \geq 0, \quad \forall i \in I, j \in J, k \in K, h \in H$$

5.2 TEMC IN ACTION – EXAMPLE 2

Optimal solution

- Open both DCs
- DCE serves customer RC & RN
- DCW serves customer RS

i	j	k	h	x_{ijk}^h	$c_{ijk}^h = c_{ij}^h$	$+ c_{jk}^h$	g_j		
SUP	**DC**	**CUS**	**Product**	**Flow Units**	**S → DC ($/unit)**	**DC → C ($/unit)**	**Handling cost ($/unit)**	**Total (w/o fixed cost)**	
FA	DCE	RC	P1	200	5	4	2	2200	(5+4+2)(200)
FA	DCE	RC	P2	150	5	4	2	1650	(5+4+2)(150)
FA	DCE	RN	P1	200	5	3	2	2000	(5+3+2)(200)
FA	DCE	RN	P2	150	5	3	2	1500	
FA	DCW	RS	P1	100	7	2	1.5	1050	
FB	DCW	RS	P1	200	6	2	1.5	1900	
FB	DCW	RS	P2	250	6	2	1.5	2375	

$f_{DCE} z_{DCE}$ $\quad$ $f_{DCW} z_{DCW}$

Minimum total costs = 12675 + 1000(1) + 1200(1) = **$14,875**

FMCG = Fast-moving customer goods (sold quickly and at a relatively low cost)

6 EXERCISE 1 – RETAIL DISTRIBUTION NETWORK

An FMCG company distributes two products, Canned Vegetables (P1) and Beverages (P2), from three suppliers through two potential distribution centers (DCs) to three retail stores. Clearly formulate the TEMC optimisation problem. Define decision variables, the objective function, and all constraints.

Supplier Capacities (units)

Supplier	P1	P2
S1	250	150
S2	200	300
S3	200	100

Distribution Center Capacities (units)

DC	Capacity	Fixed Cost ($)	Handling Cost ($/unit)
DC1	600	800	2
DC2	750	950	1.5

Customer Demands (units)

Customer	Canned Vegetables (P1)	Beverages (P2)
C1	100	100
C2	150	150
C3	200	100

Transportation Costs (Suppliers → DCs) ($ per unit)

Supplier	DC1	DC2
S1	4	5
S2	3	2
S3	5	3

Transportation Costs (DCs → Customers) ($ per unit)

DC	C1	C2	C3
DC1	2	3	4
DC2	3	2	3

6 EXERCISE 1 – RETAIL DISTRIBUTION NETWORK

Sets:

- Suppliers $i \in \{S1, S2, S3\}$
- DCs $j \in \{DC1, DC2\}$
- Customers $k \in \{C1, C2, C3\}$
- Products $h \in \{P1, P2\}$

Decision Variables:

$$x_{ij}^h, \quad y_{jk}^h, \quad z_j \in \{0, 1\}$$

Objective Function:

$$\min \sum_{i,j,h} C_{ij} x_{ij}^h + \sum_{j,k,h} C_{jk} y_{jk}^h + \sum_j (f_j z_j + g_j \sum_{k,h} y_{jk}^h)$$

Constraints:

$$\sum_j x_{ij}^h \leq P_i^h, \quad \forall i, h$$

$$\sum_i x_{ij}^h = \sum_k y_{jk}^h, \quad \forall j, h$$

$$\sum_{k,h} y_{jk}^h \leq K_j z_j, \quad \forall j$$

$$x_{ij}^h, y_{jk}^h \geq 0, \quad z_j \in \{0, 1\}$$

$$\sum_j y_{jk}^h = D_k^h, \quad \forall k, h$$

6 EXERCISE 2 – PHARMACEUTICAL NETWORK

A pharmaceutical company distributes two medicines, Medicine A (MA) and Medicine B (MB), from two factories through two potential warehouses (W1, W2) to four hospitals. Clearly formulate this scenario as a TEMC optimisation problem. Explicitly define the decision variables, objective function, and all necessary constraints.

Supplier Capacities (units)

Supplier	Medicine A (MA)	Medicine B (MB)
F1	300	200
F2	200	400

Hospital Demands (units)

Hospital	Medicine A (MA)	Medicine B (MB)
H1	100	100
H2	80	120
H3	50	150
H4	100	80

Warehouse Capacities (units)

Warehouse	Capacity	Fixed Cost ($)	Handling Cost ($/unit)
W1	400	670	1.25
W2	300	520	1.5

Transportation Costs ($ per unit)

Supplier	W1	W2
F1	4	6
F2	3	5

Warehouse	H1	H2	H3	H4
W1	3	4	2	5
W2	5	2	4	3

6 EXERCISE 2 – PHARMACEUTICAL NETWORK

Sets and Indices:

$$i \in \{F1, F2\}, \quad j \in \{W1, W2\}, \quad k \in \{H1, H2, H3, H4\}, \quad h \in \{MA, MB\}$$

Decision Variables:

$$x_{ij}^h, y_{jk}^h \geq 0, \quad z_j \in \{0, 1\}.$$

Objective Function:

$$\min \sum_{i,j,h} C_{ij} x_{ij}^h + \sum_{j,k,h} C_{jk} y_{jk}^h + \sum_j (f_j z_j + g_j \sum_{k,h} y_{jk}^h)$$

Constraints:

$$\sum_j x_{ij}^h \leq p_i^h, \quad \forall i, h$$

$$\sum_{k,h} y_{jk}^h \leq K_j z_j, \quad \forall j$$

$$\sum_j y_{jk}^h = D_k^h, \quad \forall k, h$$

$$\sum_i x_{ij}^h = \sum_k y_{jk}^h, \quad \forall j, h$$

$$z_j \in \{0, 1\}, \quad \forall j$$