SUPPLY CHAIN MODELLING AND OPTIMIZATION

TOPIC 4 SC NETWORK DESIGN UNDER UNCERTAINTY

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TOPIC 4: NETWORK DESIGN UNDER UNCERTAINTY

1 DISCOUNTED CASH FLOW (DCF)

Discounted Cash Flow (DCF) is a valuation method used to estimate the value of an investment based on its expected future cash flows. The DCF model calculates the **present value (PV)** of these cash flows, taking into account the **time value of money**.

Time value of money: Money (or CF) today are worth more than the future

$1000 today?

$1000 in 10 years time?

DCF is widely used in finance to assess the **intrinsic value** of a company or project by discounting future cash flows back to their present value.

Intrinsic value: The true, inherent, or essential worth of an object, asset, or concept—based on its fundamental characteristics or expected future benefits, rather than its current market price.

WCC represents a company's **average cost of financing from all sources**, both equity (stocks) and debt (loans or bonds), **weighted by their proportion** in the company's capital structure.

1.1 KEY COMPONENTS

Cash flows:

The net amounts of cash that are expected to be received or paid out over the investment period.

Cash **inflows** can come from profits, asset sales, etc., while **outflows** might include operating expenses, capital expenditures, etc.

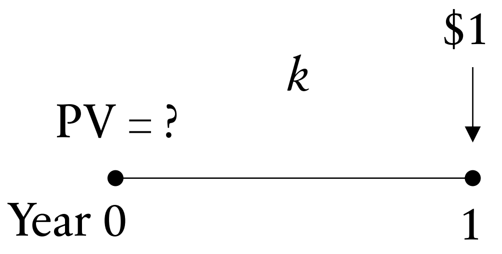
Discount rate:

The rate of return used to convert future cash flows into their present value. It reflects the risk-free rate, plus a risk premium for the uncertainty of cash flows. The Weighted Average Cost of Capital (WACC) is often used as the discount rate in DCF analysis.

Net Present Value (NPV):

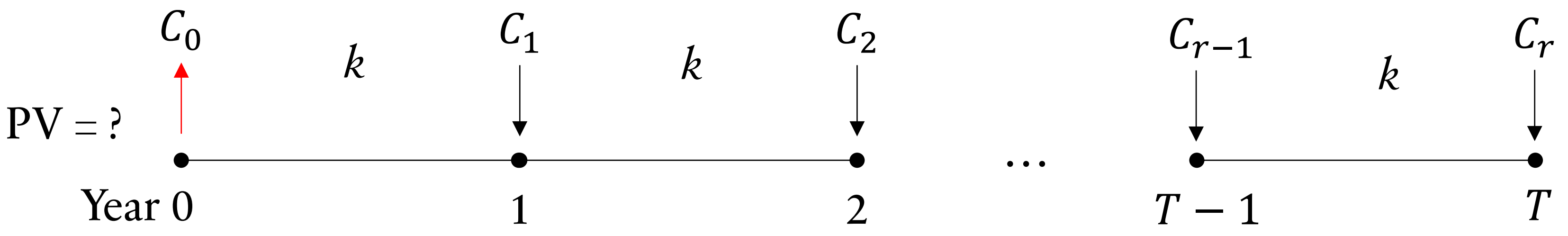
NPV is used to determine whether or not an investment, project, or business will be profitable down the line. It is the sum of all future cash flows over the investment's lifetime, discounted to the present value.

1.2 FORMULAS



$$\text{Discount factor} = \frac{1}{1+k}$$

$$\text{PV at Year 0} = (\$1)\frac{1}{1+k}$$



$$\text{NPV at Year 0} = C_0 + C_1\left(\frac{1}{1+k}\right) + C_2\left(\frac{1}{1+k}\right)^2 \ldots + C_{r-1}\left(\frac{1}{1+k}\right)^{T-1} + C_r\left(\frac{1}{1+k}\right)^T$$

$$\text{NPV} = C_0 + \sum_{t=1}^{T}\left(\frac{1}{1+k}\right)^t C_t$$

$$\text{NPV} = \sum_{t=0}^{T}\left(\frac{1}{1+k}\right)^t C_t$$

C_i is a stream of cash flows

k is the rate of return

T is the total number of period

1.2 FORMULAS

$$\text{NPV} = C_0 + \sum_{t=1}^{T} \left(\frac{1}{1+k}\right)^t C_t$$

If…	Then…	Therefore, …
NPV > 0	Investment adds value	The project may be accepted
NPV < 0	Investment subtracts value (lose money)	The project should be rejected
NPV = 0	Investment would neither add or subtract value	Decision should be based on other criteria

1.3 EXAMPLE 1

$$\text{NPV} = C_0 + \sum_{t=1}^{T} \left(\frac{1}{1+k}\right)^t C_t$$

A company is deciding whether to invest $11 million with the following estimated cash flows per year. Use 5% discount rate, determine the net present value of this investment and make decision whether the company should accept or reject this investment plan.

In accounting, negative numbers are often shown in parentheses () as an alternate way of indicating that the number is negative.

Year	Cash Flow	DCF at Year 0	
0	$ (11,000,000.00)	$ (11,000,000.00)	$-11 \times 10^6/1.05^0$
1	$ 1,000,000.00	$ 952,380.95	$1 \times 10^6/1.05^1$
2	$ 1,000,000.00	$ 907,029.48	$1 \times 10^6/1.05^2$
3	$ 4,000,000.00	$ 3,455,350.39	$4 \times 10^6/1.05^3$
4	$ 4,000,000.00	$ 3,290,809.90	$4 \times 10^6/1.05^4$
5	$ 6,000,000.00	$ 4,701,157.00	$6 \times 10^6/1.05^5$
	NPV =	$ 2,306,727.72	

The positive number of $2,306,727 indicates that the project could generate a return higher than the initial cost—a positive return on the investment. Therefore, the project may be worth making.

1.3 EXAMPLE 2

If the initial investment is \$14 million instead of \$11 million, what would be the net present value? Also, make a decision whether this project is still worth investing.

Year	Cash Flow	DCF at Year 0	
0	\$ (14,000,000.00)	\$ (14,000,000.00)	$-14 \times 10^6/1.05^0$
1	\$ 1,000,000.00	\$ 952,380.95	$1 \times 10^6/1.05^1$
2	\$ 1,000,000.00	\$ 907,029.48	$1 \times 10^6/1.05^2$
3	\$ 4,000,000.00	\$ 3,455,350.39	$4 \times 10^6/1.05^3$
4	\$ 4,000,000.00	\$ 3,290,809.90	$4 \times 10^6/1.05^4$
5	\$ 6,000,000.00	\$ 4,701,157.00	$6 \times 10^6/1.05^5$
	NPV =	**\$ \$(693,272.28)**	

The negative number of \$693,272.28 indicates that the project could generate a return lower than the initial cost—a loss on the investment. Therefore, the project is not worth investing.

1.3 EXAMPLE 3 – TARGET.COM

- Estimated demand 100,000 units per year for online orders starting immediately
- Required space 1,000 sq. ft. for every 1,000 units
- Revenue $1.22 for each unit of demand

Question:

1. How much space to lease in the next three years?
2. Should Target sign a 3-year lease contract or obtain warehousing space on the spot market each year?

Use discount factor $k = 0.1$, Target.com can choose between two options

Option 1: Spot market rate expected at $1.20 per sq.ft. per year for each of the next 3 years.

Option 2: 3-year lease contract at $1 per sq.ft. paid per year

1.3 EXAMPLE 3 – TARGET.COM

Option 1:

- Spot market rate expected at $1.20 per sq.ft. per year for each of the next 3 years.
- Required space 1,000 sq. ft. for every 1,000 units
- Estimated demand 100,000 units per year for online orders
- Revenue $1.22 for each unit of demand

Expected Annual Profit per year $(C_t) = \underbrace{(100{,}000 \times \$1.22)}_{\textbf{revenue}} - \underbrace{(100{,}000 \times \$1.20)}_{\textbf{expense}} = \$2{,}000$

Year	C_t	DCF
0	$ 2,000.00	$ 2,000.00
1	$ 2,000.00	$ 1,818.18
2	$ 2,000.00	$ 1,652.89

NPV (Option 1) = $ 5,471.07

$$\text{NPV} = C_0 + \frac{C_1}{1+k} + \frac{C_2}{(1+k)^2}$$

$$= 2{,}000 + \frac{2{,}000}{1.1} + \frac{2{,}000}{1.1^2}$$

$$= \$5{,}471.07$$

1.3 EXAMPLE 3 – TARGET.COM

Option 2:

- 100,000 sq.ft. of warehouse space are leased for the next 3 years, Target pays $1 per sq.ft. of space leased each year.
- Estimated demand 100,000 units per year for online orders
- Revenue $1.22 for each unit of demand

Expected Annual Profit per year $(C_t) = (100{,}000 \times \$1.22) - (100{,}000 \times \$1) = \$22{,}000$

revenue **expense**

Year	C_t	DCF
0	$ 22,000.00	$ 22,000.00
1	$ 22,000.00	$ 20,000.00
2	$ 22,000.00	$ 18,181.82

NPV (Option 2) = $ 60,181.82

$$\text{NPV} = C_0 + \frac{C_1}{1+k} + \frac{C_2}{(1+k)^2}$$

$$= 22{,}000 + \frac{22{,}000}{1.1} + \frac{22{,}000}{1.1^2}$$

$$= \$60{,}181.82$$

1.3 EXAMPLE 3 – TARGET.COM

Year	C_t	DCF
0	$ 2,000.00	$ 2,000.00
1	$ 2,000.00	$ 1,818.18
2	$ 2,000.00	$ 1,652.89

NPV (Option 1) = $ 5,471.07

Year	C_t	DCF
0	$ 22,000.00	$ 22,000.00
1	$ 22,000.00	$ 20,000.00
2	$ 22,000.00	$ 18,181.82

NPV (Option 2) = $ 60,181.82

The NPV of signing the lease (Option 2) is $60,181.82 - $5,471.07 = $54,710.74 higher

Assume that there is <u>no uncertainty in the demand and costs</u>,
Target.com should go with Option 2.

Although, this example is obvious from the start that Target.com should go with option 2, but it might not be the case when uncertainty is involved.

1.4 BINOMIAL REPRESENTATION OF UNCERTAINTY

A **binomial distribution** is a **discrete probability distribution** that models the number of **successes** in a fixed number of **independent** trials of a **binary experiment** — where each trial results in only one of two outcomes: **success** or **failure**.

$$f(k, n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for $k = 0,1,2, \ldots, n$, where

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Symbol	Meaning
$\Pr(X = k)$	Probability of getting exactly k successes in n trials
n	Total number of trial
k	Number of success
p	Probability of success on a single trial
$1 - p$	Probability of success on a single trial
$\binom{n}{k}$	Binomial coefficient, representing the number of combinations

1.4 BINOMIAL REPRESENTATION OF UNCERTAINTY

Example 1. Let's say you flip a coin 5 times. What's the probability of getting **exactly 3 heads**?

- $n = 5, k = 3$
- $p = 0.5$ (fair coin)

$$\Pr(X = 3) = \binom{5}{3} 0.5^3 (1 - 0.5)^{5-3} = 10 \times 0.125 \times 0.25 = 0.3125$$

There's a 31.25% chance of getting exactly 3 heads in 5 flips.

1.4 BINOMIAL REPRESENTATION OF UNCERTAINTY

Multiplicative Binomial

Given a price P in Period 0, the possible outcomes in the future periods are

Period	Outcome
0	P
1	Pu, Pd
2	$\mathrm{Pu^2, Pu, Pd^2}$
3	$\mathrm{Pu^3, Pu^2d, Pd^2u, Pd^3}$
4	$\mathrm{Pu^4, Pu^3d, Pu^2d^2, Pd^3u, Pd^4}$

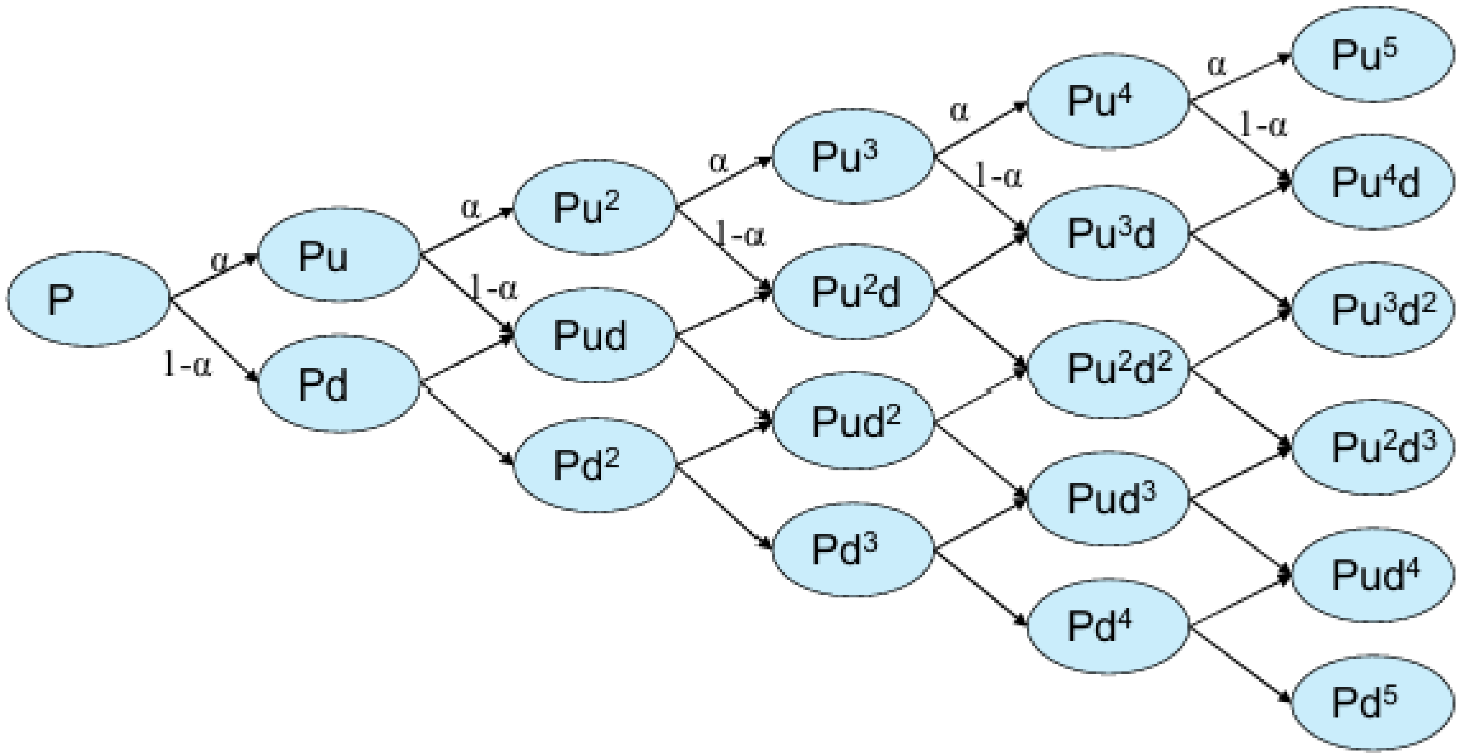
General form of all possible outcomes at period T is $Pu^t d^{(T-t)}$, $t = 0,1, \dots, T$

At instant time $t = a$, the price can move to either $Pu^{a+1} d^{(T-a)}$ with probability α

$Pu^a d^{(T-a)+1}$ with probability $1 - \alpha$

1.4 BINOMIAL REPRESENTATION OF UNCERTAINTY

Multiplicative Binomial



1.4 MULTIPLICATIVE BINOMIAL EXERCISES

1. **Given:**
   - Initial asset price P=100
   - Up factor u=1.2
   - Down factor d=0.8

Tasks:

i. Construct the binomial price tree up to **3 periods**.
ii. List all possible outcomes at t=3.
iii. Express the general formula for price at any node.

2. **Given:**
   - Initial price P=50
   - u=1.1, d=0.9
   - Risk-neutral probability α=0.6
   - T=2 periods

Tasks:

i. Calculate all possible prices at T=2.
ii. Find the probability of ending up at each price level.
iii. What is the probability that the final price is **greater than 50**?

1.4 BINOMIAL REPRESENTATION OF UNCERTAINTY

Additive Binomial

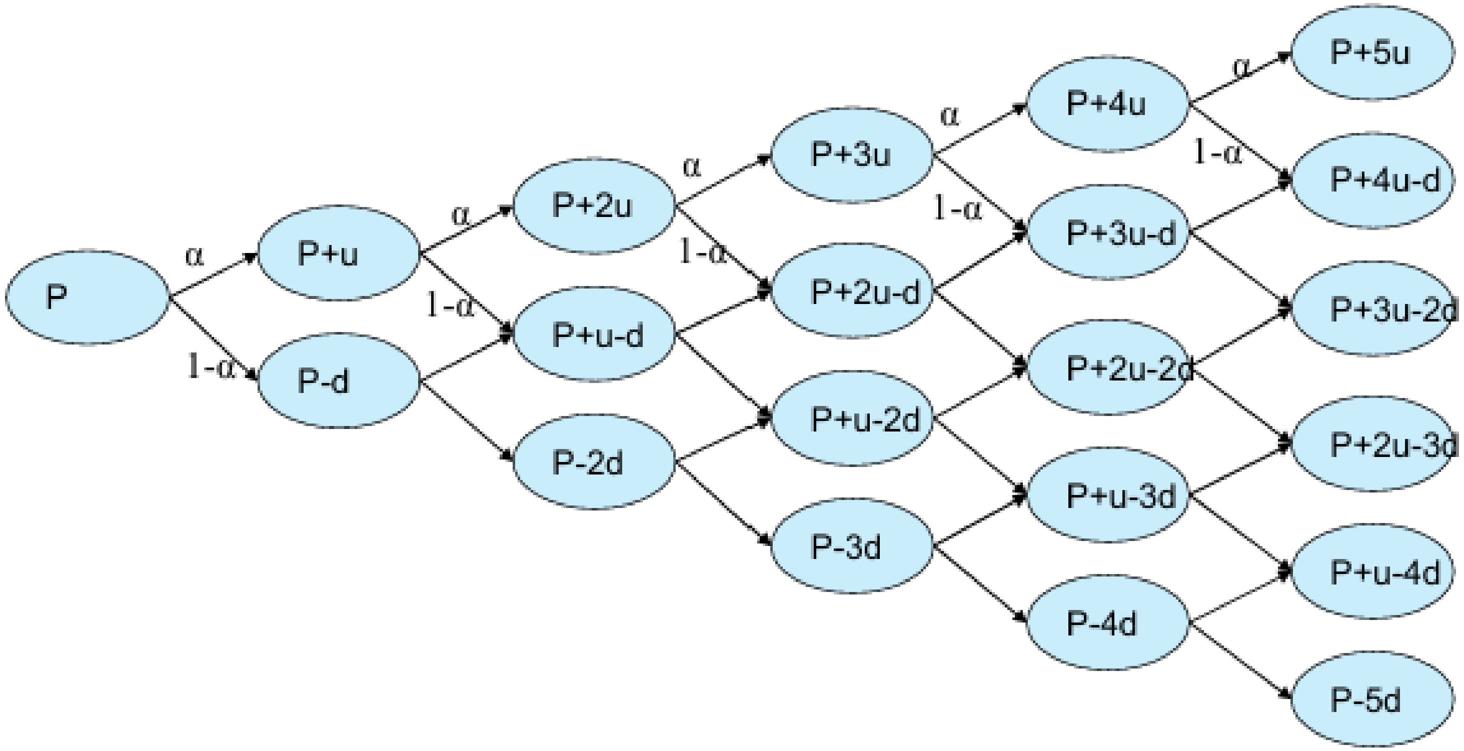
Given a price P in Period 0, the possible outcomes in the future periods are

Period	Outcome
0	P
1	$\mathrm{P + u, P - d}$
2	$\mathrm{P + 2u, P + u - d, P - 2d}$
3	$\mathrm{P + 3u, P + 2u - d, P + u - 2d, P - 3d}$
4	$\mathrm{P + 4u, P + 3u - d, P + 2u - 2d, P + u - 3d, P - 4d}$

General form of all possible outcomes at period T is $P + tu - (T - t)d, \quad t = 0,1, \dots, T$

1.4 BINOMIAL REPRESENTATION OF UNCERTAINTY

Additive Binomial



1.4 ADDITIVE BINOMIAL EXERCISES

1. **Given:**
   - Initial asset price P=50
   - Up increment u=3
   - Down decrement d=2
   - Time horizon T=3

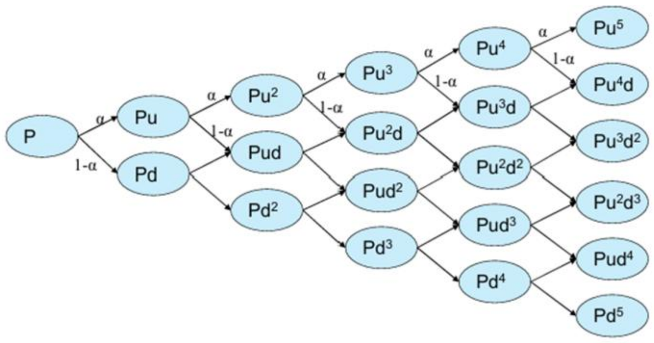
   Tasks:
   i. Construct the additive binomial price tree for 3 periods.
   ii. List all possible outcomes at T=3.
   iii. Express the general formula for price at any node.

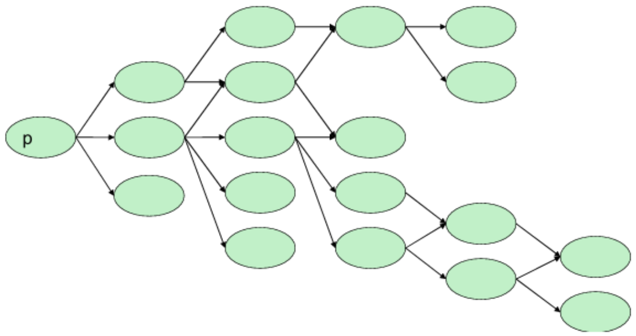
2. Using the same parameters: P = 100 and T = 2,
   - Build both the **additive** and **multiplicative** trees.
   - For additive model, use u = 5 and d = 3.
   - For multiplicative model, use u = +5%, d = -3%
   - Compare final prices.
   - What is the probability that the final price is **greater than 100**?
   - Discuss which model may be more realistic in financial contexts and why.

2 DECISION TREES

A decision tree is a decision support recursive partitioning structure that uses a tree-like model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility. It is one way to display an algorithm that only contains conditional control statements.

Binomial Tree	Decision Tree
Each node has two outcomes: up or down: (binary branching)	Each node can have multiple branches representing choices or events





Move forward in discrete time steps	Often progresses along decision-event-decision cycles
Based on risk-neutral probabilities	Includes probabilities and payoffs

2.1 EVALUATING NPV USING A DECISION TREE

Bellman's principle of optimality

"An optimal policy has the property that, regardless of the initial state, the remaining decisions must constitute an optimal policy with respect to the resulting state."

In simpler term: → Solve the problem **from the end** (**future outcomes**) **back to the beginning**.

Steps

1. **Structure the decision tree**
   - Draw nodes and terminal nodes (outcomes) connected progressively with arrows
2. **Assign payoffs and probabilities for each node**
   - For each node, calculate payoffs (or a combination of demand and price) and state the probability.
3. **Work backwards** (**Bellman's principle**)
   - From the terminal node, calculate the Expected Monetary Value (EMV) = $\sum$ (Probability × Payoff)
   - Discount EMV to Present Value (PV)
   - Continue working backward until you reach the initial decision.

2.1 EVALUATING NPV USING A DECISION TREE

Example 4. Imagine a company considering a **project** that costs $50,000. There are two scenarios:

- **High demand** (60%): earns $100,000
- **Low demand** (40%): earns $30,000
- Assume a discount rate of 10% and 1-year horizon.

Starts from the terminal node,

$$EMV = 0.6(100{,}000) + 0.4(30{,}000) = 60{,}000 + 12{,}000 = 72{,}000$$

$$PV = \frac{72{,}000}{1.1} = 65{,}455$$

$$NPV = PV - \text{Initial cost} = 65{,}455 - 50{,}000 = \mathbf{15,455}$$

2.1 EVALUATING NPV USING A DECISION TREE

Example 5. Imagine a company considering a **project** that costs $50,000.

In **Year 1**,

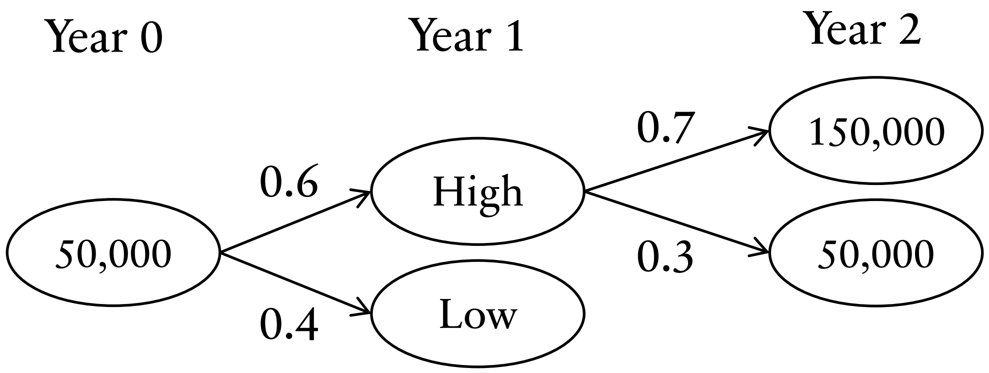
- High demand (60%): the company can invest **$20,000 more** to expand.
- Low demand (40%): the project ends; no further payoff.

In **Year 2**, expansion leads to:

- $150,000 (70%)
- $50,000 (30%)

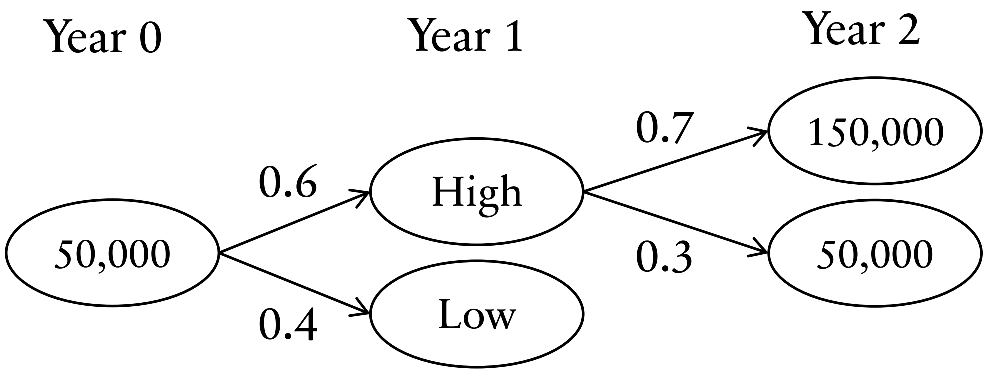
Note. For this example, the cashflows are realised at Year 2.

Use the discount rate of 10%.



2.1 EVALUATING NPV USING A DECISION TREE

Example 5.



Note. The cashflows are realised at Year 2.

Year 2: $EMV_{Year2} = 0.7(150{,}000) + 0.3(50{,}000) = 105{,}000 + 15{,}000 = 120{,}000$

Discount to Year 1, $PV_{Year1} = \dfrac{120{,}000}{1.1} = 109{,}090.91$

Net payoff at Year 1 (if expand) = $109{,}090.91 - 20{,}000 = 89{,}090.91$

Year 1: $EMV_{Year1} = 0.6(89{,}090.91) + 0.4(0) = 53{,}454.55$

Discount to Year 0, $PV_{Year0} = \dfrac{53{,}454.55}{1.1} = 48{,}595.04$

$NPV = 48{,}595.04 - 50{,}000 = \mathbf{-1{,}404.96}$

2.1 EVALUATING NPV USING A DECISION TREE

Example 6. Imagine a company considering a **project** that costs $50,000.

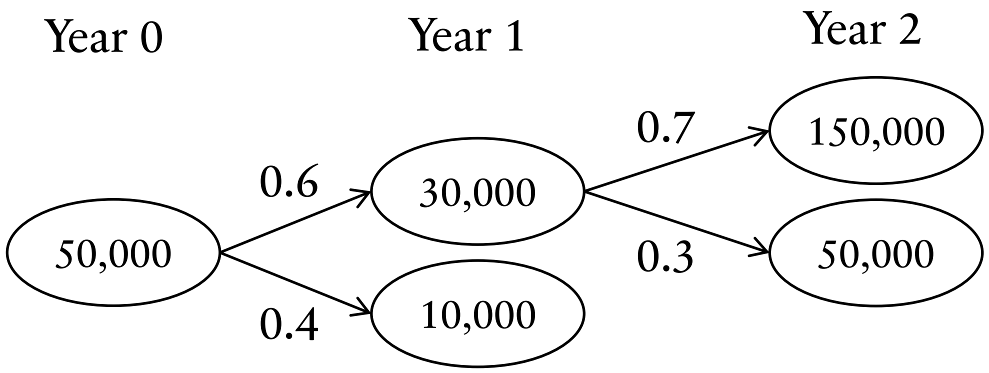
In **Year 1**,

- High demand (60%):
  - Earns $30,000 immediately
  - the company can invest **$20,000 more** to expand.
- Low demand (40%):
  - Earns $10,000 immediately
  - the project terminates.

In **Year 2**, expansion leads to:

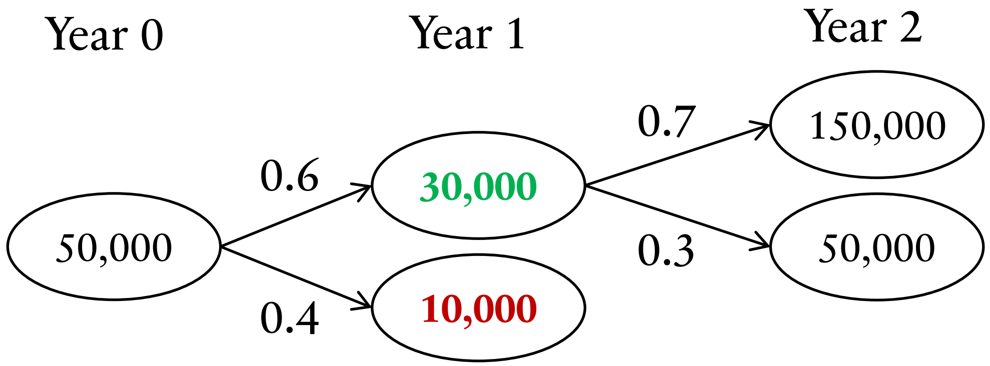
- $150,000 (70%)
- $50,000 (30%)

Use the discount rate of 10%.



2.1 EVALUATING NPV USING A DECISION TREE

Example 6.



Year 2: $EMV_{Year2} = 0.7(150{,}000) + 0.3(50{,}000) = 105{,}000 + 15{,}000 = 120{,}000$

Discount to Year 1, $$PV_{Year1} = \frac{120{,}000}{1.1} = 109{,}090.91$$

Net payoff at Year 1 (if expand) = $109{,}090.91 - 20{,}000 + \mathbf{30{,}000} = 119{,}090.91$

Year 1: $EMV_{Year1} = 0.6(119{,}090.91) + 0.4(\mathbf{10{,}000}) = 75{,}454.55$

Discount to Year 0, $$PV_{Year0} = \frac{75{,}454.55}{1.1} = 68{,}595.95$$

$$NPV = 68{,}595.95 - 50{,}000 = \mathbf{18{,}595.95}$$

2.1 EVALUATING NPV USING A DECISION TREE

Let's revisit **Example 3 – Target.com**

- The long-term lease is currently cheaper than the spot market rate of warehouse space.
- The manager anticipates **uncertainty in demand and spot prices** for warehouse space over the coming three years.

Long-term lease could…

- be sufficient and consistently cheaper option.
- **be insufficient** if demand is higher than anticipated.
- **go unused** if demand lower than anticipated.
- **end up being more expensive** if future spot market prices fall

Spot market rates also fluctuate with demand…

2.1 EVALUATING NPV USING A DECISION TREE

Identify all the options:

Option 1: Get **all** warehousing space from the **spot market** as needed.

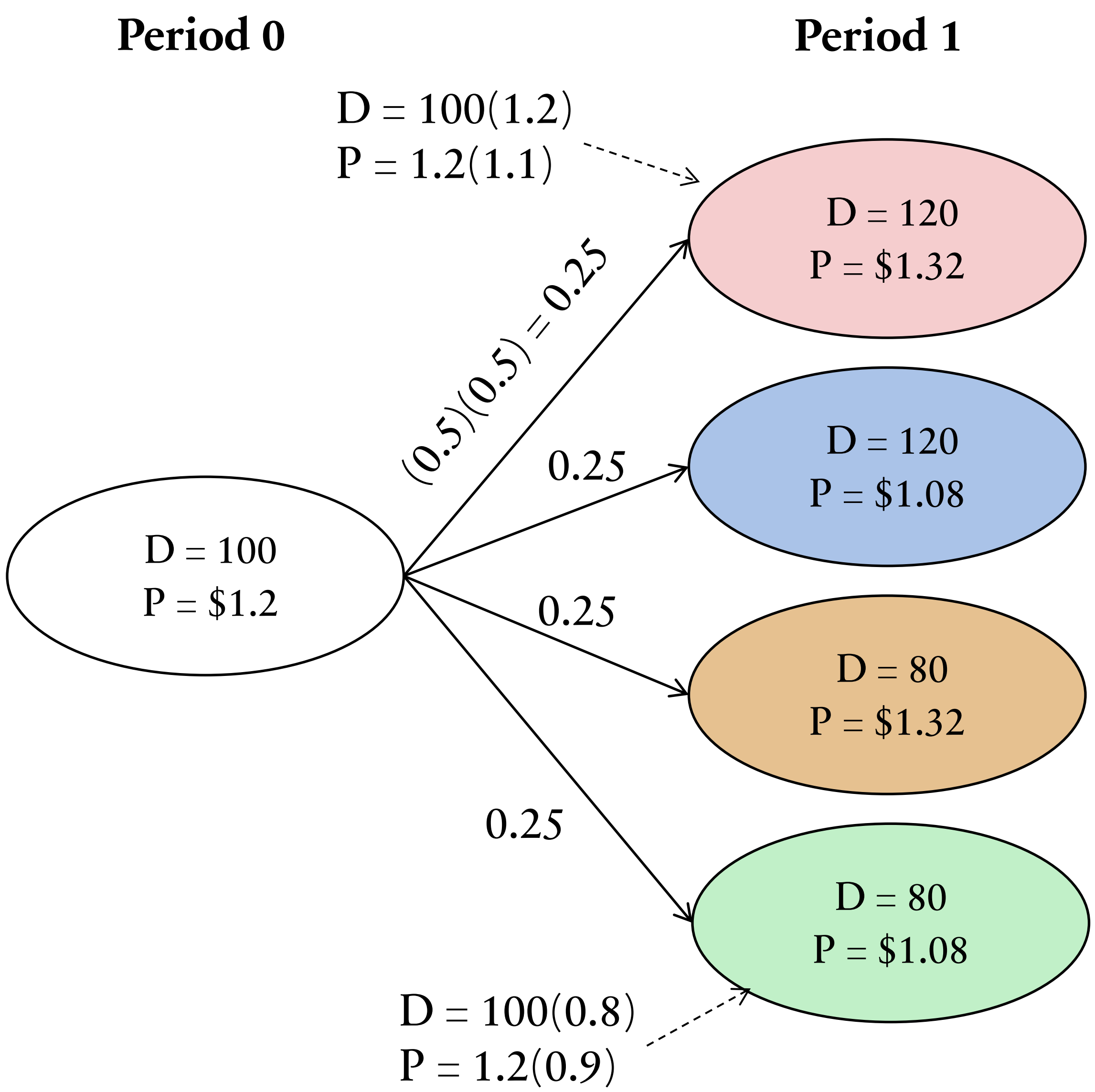
Option 2: Sign a **three-year lease** for a fixed amount of warehouse space (100,000 sq.ft.) and get additional requirements from the spot market.

Option 3: Sign a **flexible lease** with a minimum charge that allows **variable usage** of warehouse space up to a limit with additional requirement from the spot market.

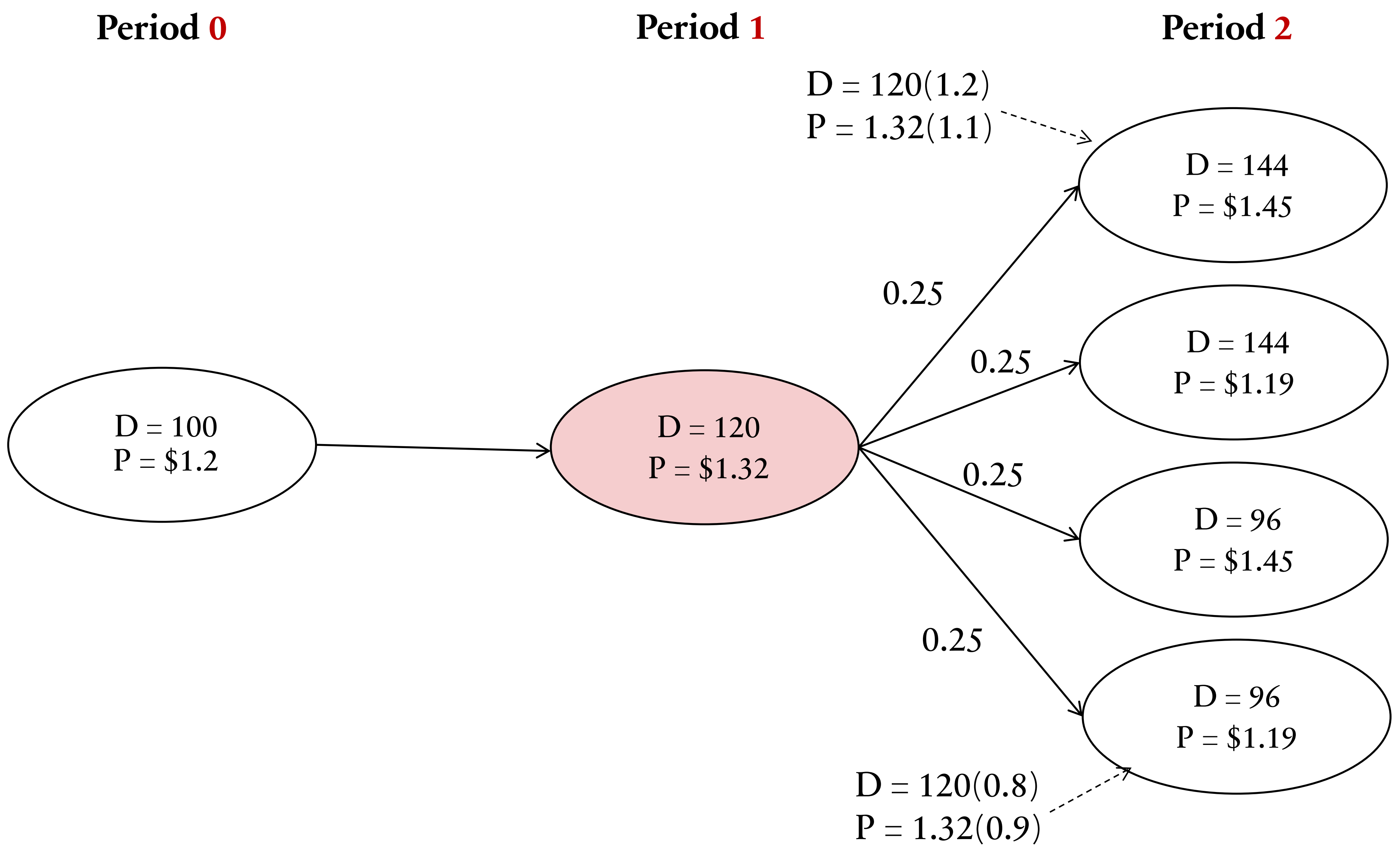
Information:

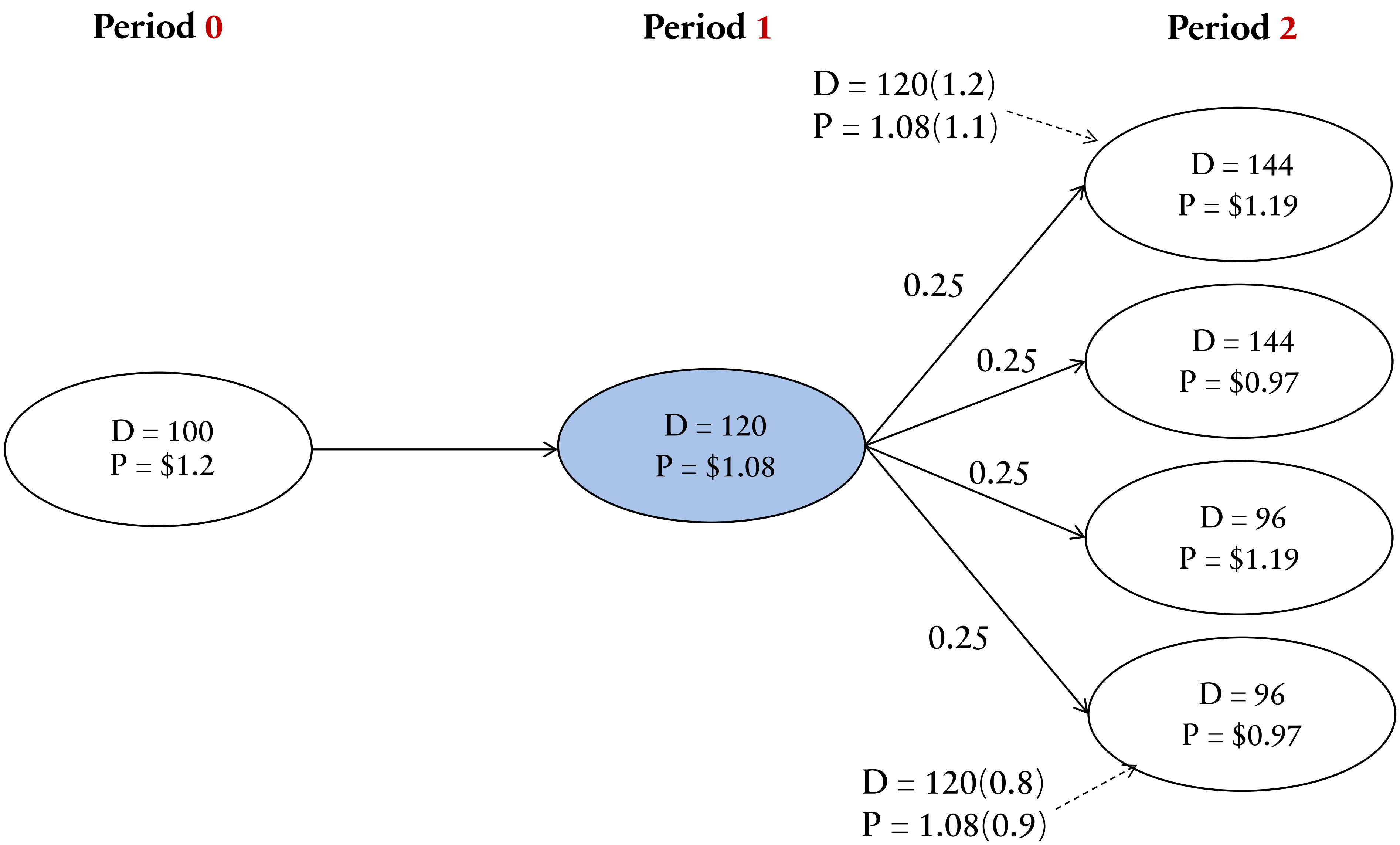
- 1000 sq.ft is required for every 1000 units of demand
- Demand (D) = 100,000 units per year, which can go up or down by 20% with the probability = 0.5
- Lease price = $1 / sq.ft / year
- Spot market price (P) = $1.2 / sq.ft / year, which can go up or down by 10% with the probability = 0.5
- Revenue = $1.22 / unit
- Prices of warehouse space and demand for product fluctuate **independently**; thus, $P(A \cap B) = P(A)P(B)$.
- k = 10%

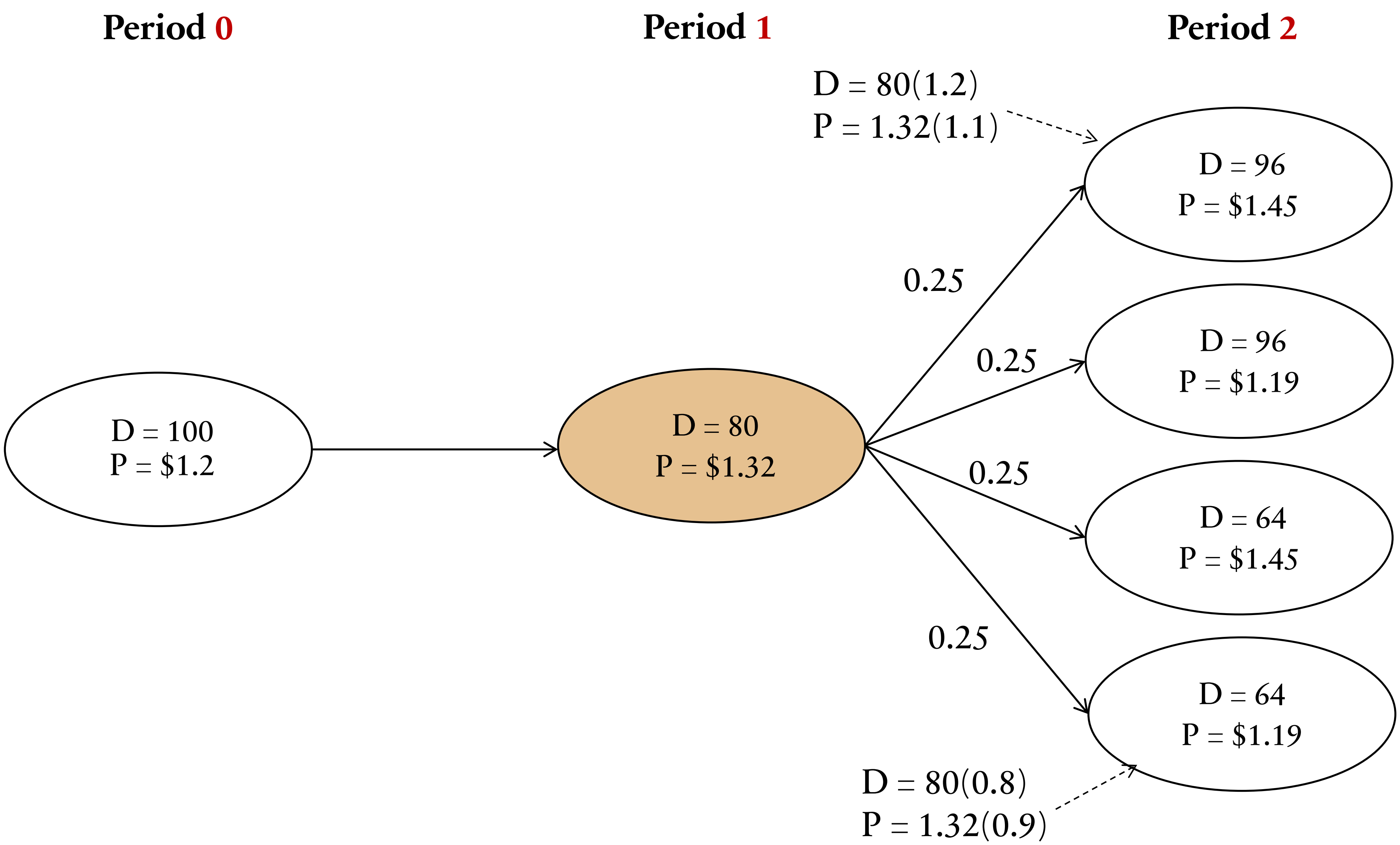
1. Construct the decision tree

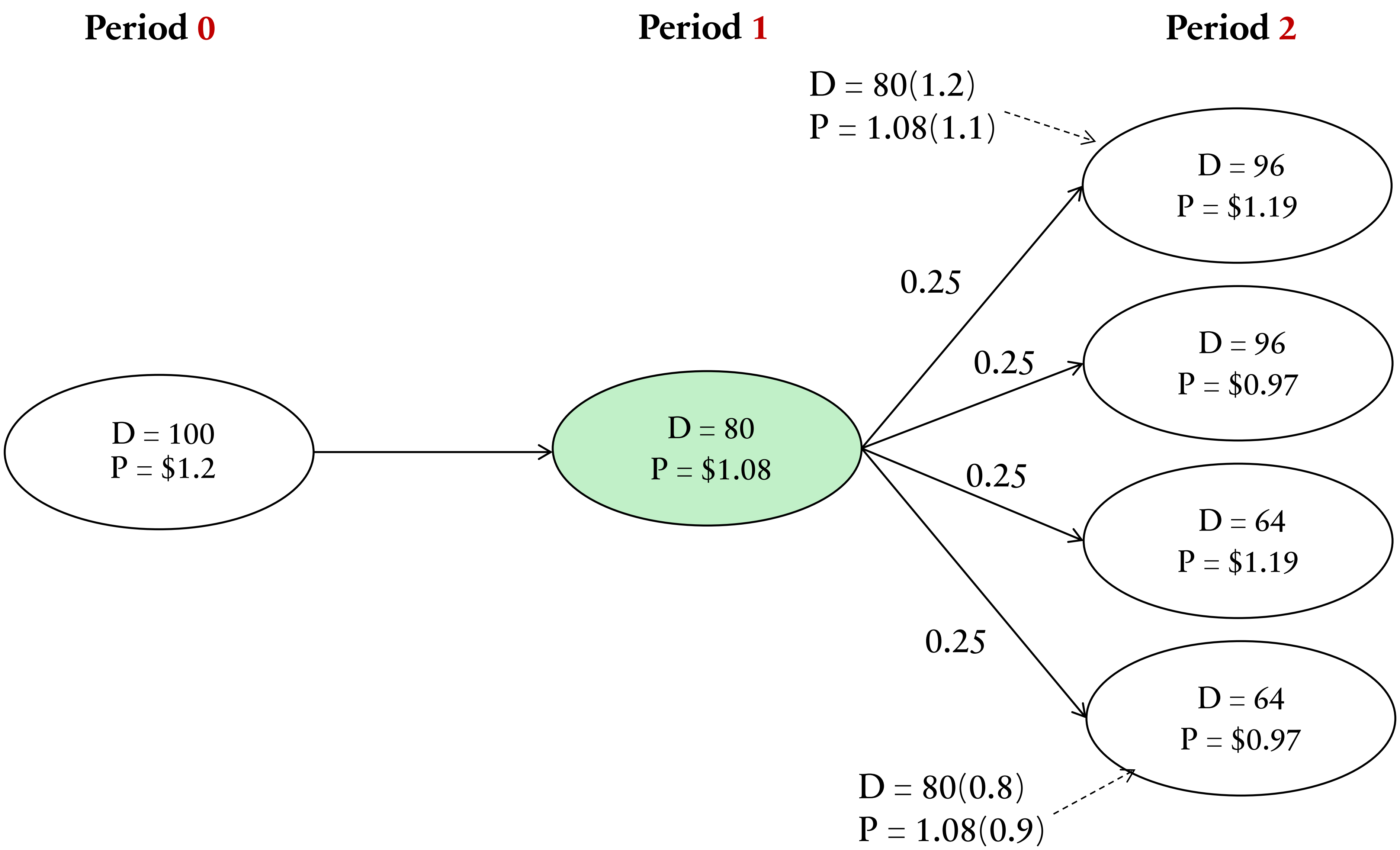


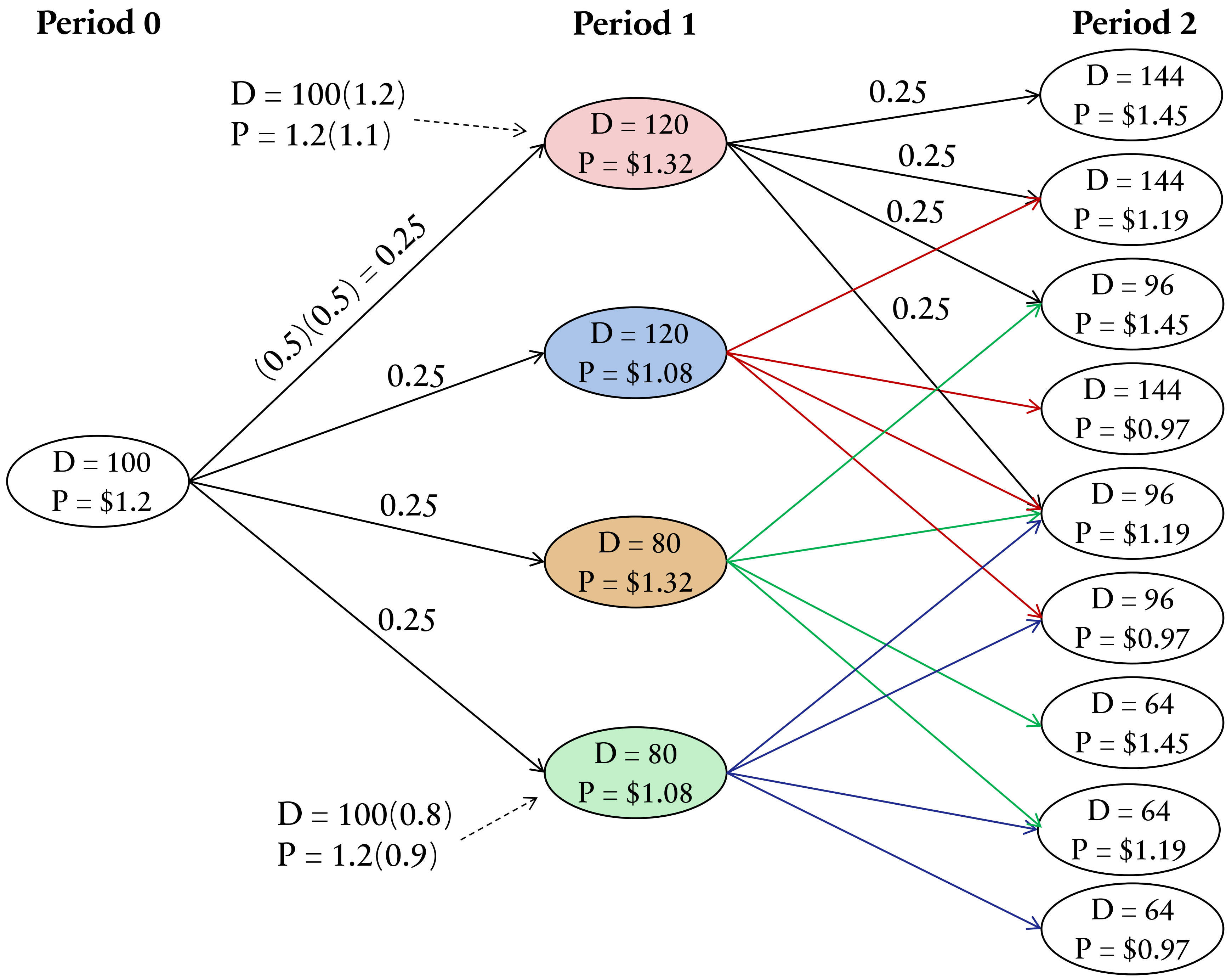
Let's consider each node at Period 1


Period 0
Period 1
Period 2
D = 120(1.2)
P = 1.32(1.1)
D = 144
P = $1.45
0.25
D = 144
P = $1.19
0.25
D = 100
P = $1.2
D = 120
P = $1.32
0.25
D = 96
P = $1.45
0.25
D = 96
P = $1.19
D = 120(0.8)
P = 1.32(0.9)


Period 0
Period 1
Period 2
D = 100
P = $1.2
D = 120
P = $1.08
0.25
0.25
0.25
0.25
D = 120(1.2)
P = 1.08(1.1)
D = 144
P = $1.19
D = 144
P = $0.97
D = 96
P = $1.19
D = 96
P = $0.97
D = 120(0.8)
P = 1.08(0.9)


Period 0
Period 1
Period 2
D = 80(1.2)
P = 1.32(1.1)
D = 96
P = $1.45
0.25
0.25
D = 96
P = $1.19
D = 100
P = $1.2
D = 80
P = $1.32
0.25
D = 64
P = $1.45
0.25
D = 64
P = $1.19
D = 80(0.8)
P = 1.32(0.9)


Period 0
Period 1
Period 2
D = 80(1.2)
P = 1.08(1.1)
D = 96
P = $1.19
0.25
0.25
D = 96
P = $0.97
D = 100
P = $1.2
D = 80
P = $1.08
0.25
D = 64
P = $1.19
0.25
D = 64
P = $0.97
D = 80(0.8)
P = 1.08(0.9)


Period 0
Period 1
Period 2
D = 100
P = $1.2
D = 100(1.2)
P = 1.2(1.1)
(0.5)(0.5) = 0.25
0.25
0.25
0.25
D = 100(0.8)
P = 1.2(0.9)
D = 120
P = $1.32
D = 120
P = $1.08
D = 80
P = $1.32
D = 80
P = $1.08
0.25
0.25
0.25
0.25
D = 144
P = $1.45
D = 144
P = $1.19
D = 96
P = $1.45
D = 144
P = $0.97
D = 96
P = $1.19
D = 96
P = $0.97
D = 64
P = $1.45
D = 64
P = $1.19
D = 64
P = $0.97

2.1 EVALUATING NPV USING A DECISION TREE

Option 1: Get **all** warehousing space from the **spot market** as needed.

1. Calculate payoff at Period 2

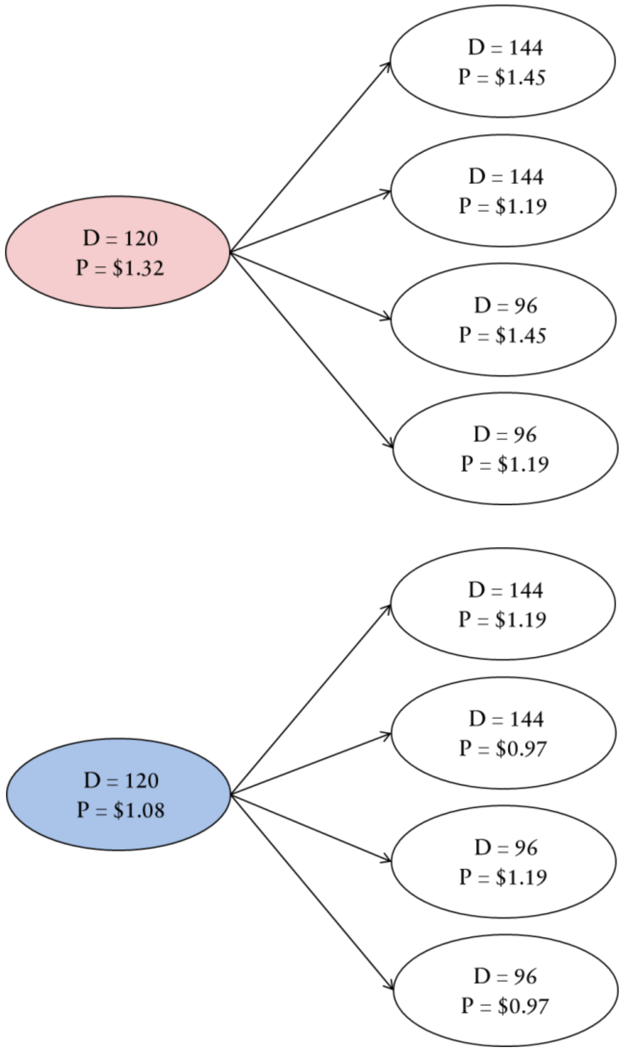
From the Period 2 node		D × 1.22	D × P	Revenue - Cost
Demand (D)	**Spot Price (P)**	**Revenue**	**Cost**	**Profit (Payoff)**
144000	1.45	175680	208800	-33120
144000	1.19	175680	171360	4320
144000	0.97	175680	139680	36000
96000	1.45	117120	139200	-22080
96000	1.19	117120	114240	2880
96000	0.97	117120	93120	24000
64000	1.45	78080	92800	-14720
64000	1.19	78080	76160	1920
64000	0.97	78080	62080	16000

$$EMV = \sum (\text{Probability} \times \text{Payoff})$$

2.1 EVALUATING NPV USING A DECISION TREE

2. Calculate EMV at Period 2

Demand (D)	Spot Price (P)	Profit (Payoff)	EV @ Period 2
144000	1.45	-33120	-8280
144000	1.19	4320	1080
144000	0.97	36000	9000
96000	1.45	-22080	-5520
96000	1.19	2880	720
96000	0.97	24000	6000
64000	1.45	-14720	-3680
64000	1.19	1920	480
64000	0.97	16000	4000



-33120(0.25) = - 8280

4320(0.25) = 1080

-22080(0.25) = -5520

2880(0.25) = 720

EMV @ Period 1

-12000

2.1 EVALUATING NPV USING A DECISION TREE

3. Calculate Payoff at Period 1

From the Period 1 node		From step 2	$\frac{\text{EMV @ Period 1}}{1.1}$	D × 1.22	D × P	PV @ Period 1 + Revenue - Cost
Demand (D)	**Spot Price (P)**	**EMV @ Period 1**	**PV @ Period 1**	**Revenue**	**Cost**	**Profit (Payoff)**
120000	1.32	-12000	-10909.09	146400	158400	-22909.09
120000	1.08	16800	15272.73	146400	129600	32072.73
80000	1.32	-8000	-7272.73	97600	105600	-15272.73
80000	1.08	11200	10181.82	97600	86400	21381.82

4. Calculate NPV at Period 0

From the Period 0 node		From step 3	$\frac{\text{EMV @ Period 0}}{1.1}$	D × 1.22	D × P	PV @ Period 0 + Revenue - Cost
Demand (D)	**Spot Price (P)**	**EMV @ Period 0**	**PV @ Period 0**	**Revenue**	**Cost**	**Profit (Payoff)**
100000	1.2	3818.18	3471.07	122000	120000	5471.07

2.1 EVALUATING NPV USING A DECISION TREE

Option 2: Sign a **3-year lease for 100,000 sq.ft.** for $1/sq.ft. and obtain extra on spot market if needed.

1. Calculate payoff at Period 2

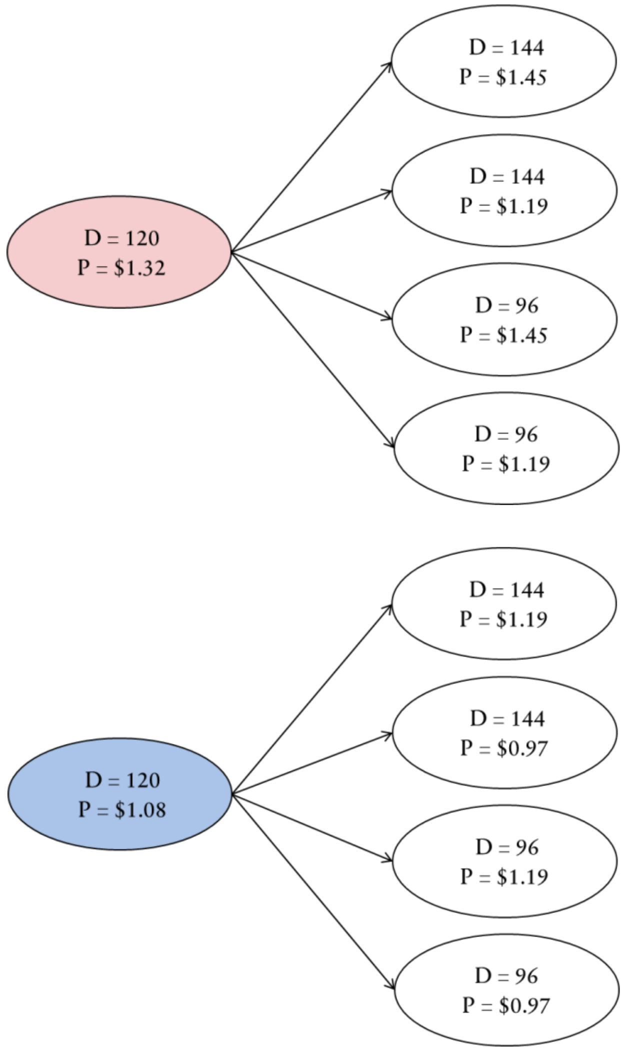
		$D \times 1.22$	$100000 + \max(D - 100000, 0) \times P$	Revenue - Cost
Demand (D)	**Spot Price (P)**	**Revenue**	**100k Lease + Extra Warehouse Cost**	**Profit (Payoff)**
144000	1.45	175680	163800	11880
144000	1.19	175680	152360	23320
144000	0.97	175680	142680	33000
96000	1.45	117120	100000	17120
96000	1.19	117120	100000	17120
96000	0.97	117120	100000	17120
64000	1.45	78080	100000	-21920
64000	1.19	78080	100000	-21920
64000	0.97	78080	100000	-21920

$$EMV = \sum (\text{Probability} \times \text{Payoff})$$

2.1 EVALUATING NPV USING A DECISION TREE

2. Calculate EMV at Period 2

Demand (D)	Spot Price (P)	Profit (Payoff)	EV @ Period 2
144000	1.45	11880	2970
144000	1.19	23320	5830
144000	0.97	33000	8250
96000	1.45	17120	4280
96000	1.19	17120	4280
96000	0.97	17120	4280
64000	1.45	-21920	-5480
64000	1.19	-21920	-5480
64000	0.97	-21920	-5480



11880(0.25) = 2970

23320(0.25) = 5830

17120(0.25) = 4280

17120(0.25) = 4280

EMV @ Period 1

17360

2.1 EVALUATING NPV USING A DECISION TREE

$$100000 + \max(D - 100000, 0) \times P$$

3. Calculate Payoff at Period 1

From the Period 1 node		From step 2	$\frac{\text{EMV @ Period 1}}{1.1}$	$D \times 1.22$		PV @ Period 1 + Revenue - Cost
Demand (D)	**Spot Price (P)**	**EMV @ Period 2**	**PV @ Period 1**	**Revenue**	**Cost**	**Profit (Payoff)**
120000	1.32	17360	15781.82	146400	126400	35781.82
120000	1.08	22640	20581.82	146400	121600	45381.82
80000	1.32	-2400	-2181.82	97600	100000	-4581.82
80000	1.08	-2400	-2181.82	97600	100000	-4581.82

4. Calculate NPV at Period 0

From the Period 0 node		From step 3	$\frac{\text{EMV @ Period 0}}{1.1}$	$D \times 1.22$		PV @ Period 0 + Revenue - Cost
Demand (D)	**Spot Price (P)**	**EMV @ Period 0**	**PV @ Period 0**	**Revenue**	**Cost**	**Profit (Payoff)**
100000	1.2	18000	16363.64	122000	100000	38363.64

2.1 EVALUATING NPV USING A DECISION TREE

Option 3: Sign a **flexible lease** with a minimum charge of $10,000, the company is able to

- Have the flexibility of using between 60,000 and 100,000 sq.ft of warehouse space at $1 / sq. ft. / year.
- Pay $60,000 / year for the first 60,000 sq.ft. and can then use up to another 40,000 sq.ft. on demand of $1 / sq.ft.

1-2. Calculate payoff and EV at Period 2

$\min(100000, D) \times \$1 + \max(D - 100000, 0) \times P$

$\text{Profit} \times 0.25$

Demand (D)	Spot Price (P)	Revenue	Cost	Profit (Payoff)	EV @ Period 2
144000	1.45	175680	163800	11880	2970
144000	1.19	175680	152360	23320	5830
144000	0.97	175680	142680	33000	8250
96000	1.45	117120	96000	21120	5280
96000	1.19	117120	96000	21120	5280
96000	0.97	117120	96000	21120	5280
64000	1.45	78080	64000	14080	3520
64000	1.19	78080	64000	14080	3520
64000	0.97	78080	64000	14080	3520

2.1 EVALUATING NPV USING A DECISION TREE

$$\min(100000, D) \times \$1 + \max(D - 100000, 0) \times P$$

3. Calculate Payoff at Period 1

From the Period 1 node (Demand, Spot Price); From step 2 (EMV @ Period 2); PV @ Period 1 = $\frac{\text{EMV @ Period 1}}{1.1}$; Revenue = $D \times 1.22$; Cost = $\min(100000, D) \times \$1 + \max(D - 100000, 0) \times P$; Profit = PV @ Period 1 + Revenue - Cost

Demand (D)	Spot Price (P)	EMV @ Period 2	PV @ Period 1	Revenue	Cost	Profit (Payoff)
120000	1.32	19360	17600.00	146400	126400	37600.00
120000	1.08	24640	22400.00	146400	121600	47200.00
80000	1.32	17600	16000.00	97600	80000	33600.00
80000	1.08	17600	16000.00	97600	80000	33600.00

4. Calculate NPV at Period 0

From the Period 0 node (Demand, Spot Price); From step 3 (EMV @ Period 0); PV @ Period 0 = $\frac{\text{EMV @ Period 0}}{1.1}$; Revenue = $D \times 1.22$; Cost = 100,000 + 10,000 (upfront payment); Profit = PV @ Period 0 + Revenue - Cost

Demand (D)	Spot Price (P)	EMV @ Period 0	PV @ Period 0	Revenue	Cost	Profit (Payoff)
100000	1.2		34545.45	122000	110000	

2.1 EVALUATING NPV USING A DECISION TREE

Comparing all options

	Option 0 (ignore uncertainty)	Option 1 (spot market only)	Option 2 (lease 100k + spot)	Option 3 (Flexible lease + spot)
NPV	60181.82	5471.07	38363.64	46545.45

- Option 0 has highest NPV as it underestimates all the fluctuations in demands and costs.
- Option 3 yields highest NPV and would be the best option for Target.com.

2.2 PRACTICE QUESTION

A food distribution company must decide how to procure **cold storage space** for the next **2 years**. It has **three options**:

- **Option 1**: Purchase all cold storage space as needed from the **spot market**.
- **Option 2**: Sign a **2-year fixed contract** to lease a predetermined amount of storage, and fulfill any extra need through the spot market.
- **Option 3**: Sign a **flexible-use lease**, which includes a minimum committed space and allows expansion up to a cap, with excess paid at spot prices.

2.2 PRACTICE QUESTION

Information:

- **Cold storage requirement**: 1 sq.ft. per 500 frozen items.
- **Expected base demand**: 250,000 frozen items per year.
- Demand may rise or fall by **25%** annually with **equal probability** (0.5).
- Spot market price: **$2.00/sq.ft/year**, may rise or fall by **15%**, also with **equal probability**.
- Fixed lease price: **$1.75/sq.ft/year**
- Revenue: **$2.50 per frozen item**
- Flexible lease of **$20,000 upfront**:
  - Covers up to 250,000 items (i.e., 500 sq.ft).
  - Charged at $1.75/sq.ft.
  - Additional needs paid at spot price.
- Spot price and demand are **independent**.
- **Discount rate** k = 8%

Your Tasks:

1. **List all possible combinations** of demand and spot prices.
2. **Calculate total storage needs** (in sq.ft) for each demand level.
3. **Compute annual profit** under each option across all scenarios.
4. **Calculate expected NPV** over the 2-year period for each option.
5. **Determine the best option** using decision tree logic (Bellman's Principle).