SUPPLY CHAIN MODELLING AND OPTIMIZATION

TOPIC 5 FORECASTING TECHNIQUES

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TOPIC 5 FORECASTING TECHNIQUES

1. Introduction to forecasting techniques in supply chain management
2. Regression Analysis
3. Moving Average Model
4. Weighted Moving Average Model
5. Exponential Smoothing Model
6. Comparative insights
7. Practice Question

1 INTRODUCTION

Forecasting is the process of making predictions based on past and present data.

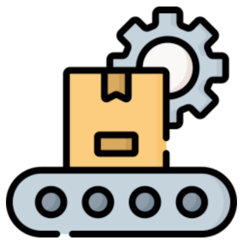
Purpose in Supply Chain:

- Align supply with anticipated demand
- Reduce inventory holding costs
- Minimise stockouts and overstock
- Support production planning, procurement, and logistics

Forecasting

Planning

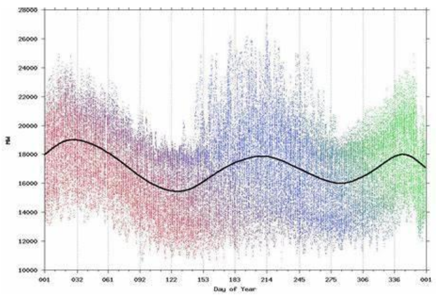
Procurement

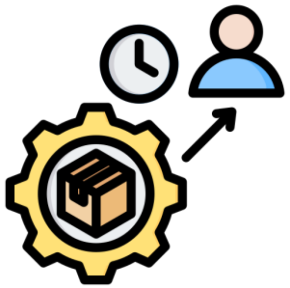


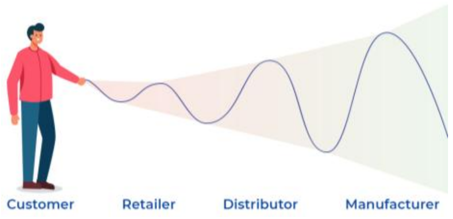
Production

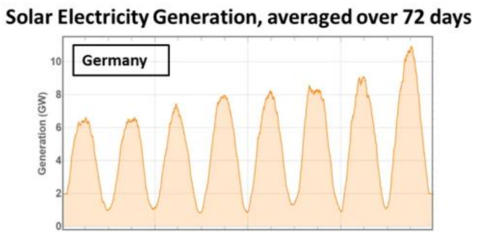
Delivery

1 INTRO – FORECASTING CHALLENGES IN SC









Demand variability

The fluctuations or changes in customer demand for a product or service over a specific period, which could cause disruptions in the supply chain.

Lead time fluctuations

The inconsistency or unpredictability in the duration it takes for tasks or materials to traverse the supply chain.

Data quality issues

An intolerable defect in a dataset that reduces its reliability and trustworthiness. The most common issues are incomplete and inconsistent data.

Bullwhip effect

A SC phenomenon where a change in demand at the retail level leads to increasingly larger fluctuations in demand at the wholesaler and manufacturer levels.

Seasonality and trend shifts

The periodic fluctuations that occur at specific regular intervals, such as weekly, monthly, or quarterly, often influenced by seasonal factors.

2 REGRESSION ANALYSIS

- A statistical method for modelling the relationship between a dependent variable and one or more independent variables.
- Commonly used to identify trends and make forecasts when **data shows a linear pattern**.

Equation of the Best-Fit Line

$$\hat{y} = a + bx$$

- $\hat{y}$ = predicted value
- x = independent variable
- a = intercept
- b = slope

$$b = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$$

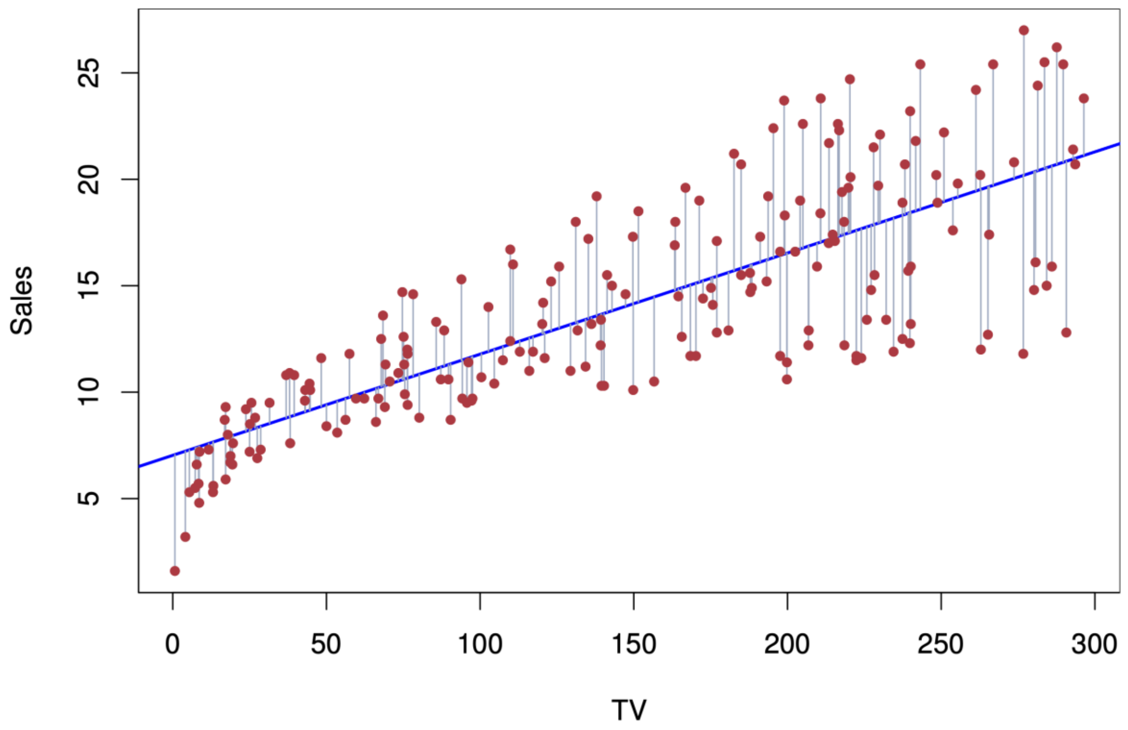


Figure 1. Forecasting of weekly sales based on TV promotional spend

2 REGRESSION ANALYSIS

The Least Squares Method

In linear regression, the **best-fit line** is the straight line that most accurately represents the relationship between the independent variable (input) and the dependent variable (output). It is the line that **minimises the difference between the actual data points and the predicted values** from the model.

These differences are called **residuals.**

$$\text{Residual} = y_i - \hat{y}_i$$

- y = the actual observed value
- $\hat{y}_i$ = predicted value

The least squares method **minimises the sum of the squared residuals** (**SSE**):

$$SSE = \sum(y_i - \hat{y}_i)^2$$

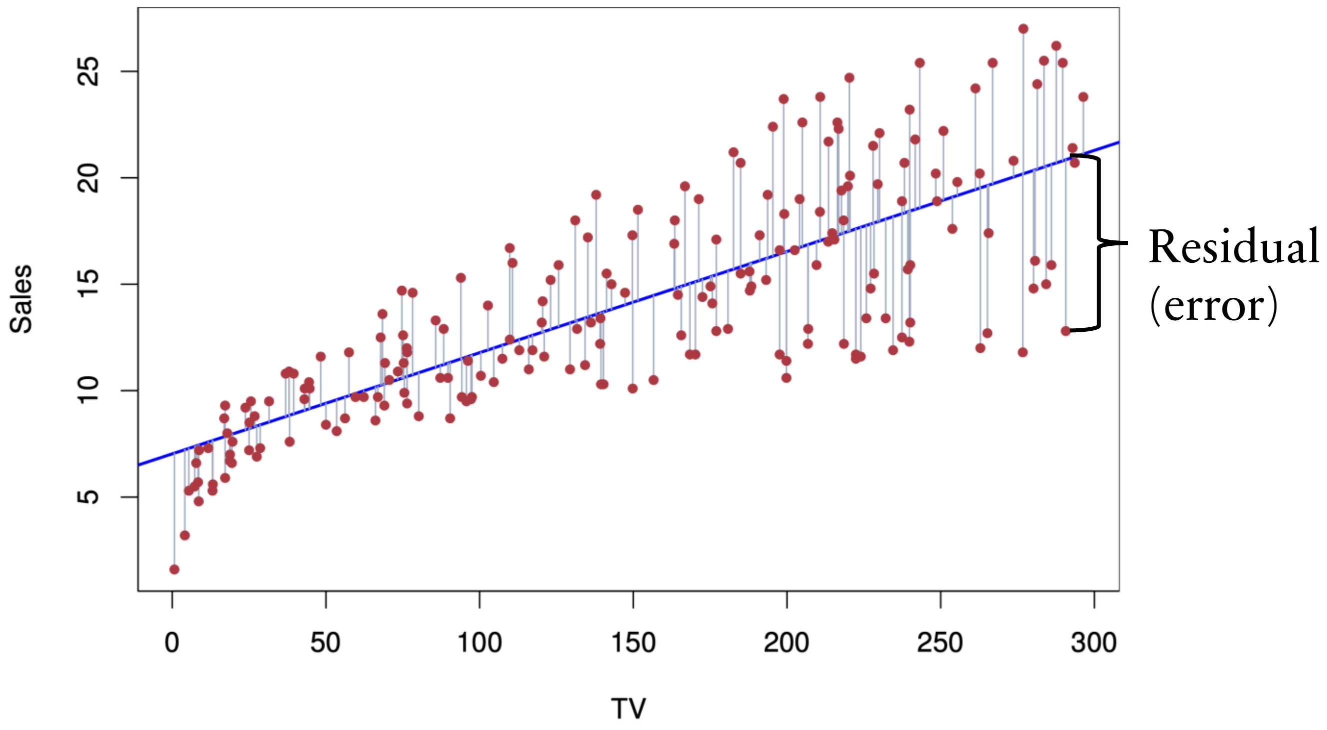


Figure 1. Forecasting of weekly sales based on TV promotional spend

2 REGRESSION ANALYSIS

$$\hat{y} = a + bx$$

Curve Fitting Given n observations (X_i, Y_i), we can fit a line to the overall pattern of these data points.

From the Least Squares Method, we can calculate the slope b and intercept a using the following formula:

$$b = \frac{\sum XY - n\bar{x}\bar{y}}{\sum X^2 - n\bar{x}^2} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}, \qquad a = \bar{y} - b\bar{x},$$

$$\bar{x} = \frac{\sum X}{n}, \qquad \bar{y} = \frac{\sum Y}{n}$$

Interpretation

- **Slope (b):** The slope of the best-fit line indicates how much the dependent variable (y) changes with each unit change in the independent variable (x). For example, if the slope is 5, it means that for every 1-unit increase in x, the value of y increases by 5 units.

- **Intercept (a):** The intercept represents the predicted value of y when $x = 0$. It's the point where the line crosses the y-axis.

2 REGRESSION ANALYSIS – EXAMPLE

Predict the demand (Y) based on week number (X) using simple linear regression.

Step 1: Raw Data

Week (X)	Demand (Y)
1	120
2	150
3	170
4	200
5	220

Step 2: Compute Intermediate Values

	X	Y	X^2	XY
	1	120	1	120
	2	150	4	300
	3	170	9	510
	4	200	16	800
	5	220	25	1100
$\sum$	**15**	**860**	**55**	**2830**

2 REGRESSION ANALYSIS – EXAMPLE

Calculate the predicted demand for each week and determine the corresponding SSE.

Step 3: Compute $\hat{Y}$ and $(Y_i - \hat{Y})^2$

	X	Y	$\hat{Y}$	$(Y_i - \hat{Y})$	$(Y_i - \hat{Y})^2$
	1	120			
	2	150			
	3	170			
	4	200	197	3	9
	5	220	222	-2	4
Σ	15	860	863		

$$SSE = \sum(y_i - \hat{y}_i)^2$$

2 REGRESSION ANALYSIS – EXERCISES

Hint: For each dataset, compute X^2 and XY, $\sum X$, $\sum Y$, $\sum X^2$, $\sum XY$, slope b, and intercept a. Then, write out the regression equation.

Exercise 1: Fit a linear regression model to predict Sales (Y) from Ad Spend (X). Use the equation to predict sales when ad spend = 275.

Ad Spend	100	150	200	250	300
Sales	20	25	30	32	35

Exercise 2: Fit a regression model to forecast production time based on the number of units.

Units	10	20	30	40	50
Time (hrs)	50	45	40	35	30

Exercise 3: Fit a linear regression line. Then, estimate how many visitors the site will get if 6 blog posts are published in a week.

Post Published	2	4	3	5	1
Visitors	120	190	170	220	100

2 REGRESSION ANALYSIS – EXERCISES

Exercise 1

X	Y	X^2	XY
Σ			

Exercise 2

X	Y	X^2	XY
Σ			

Exercise 3

X	Y	X^2	XY
Σ			

3 MOVING AVERAGE MODEL

- A moving average (MA) forecasts future values by averaging a fixed number of the most recent actual data points.
- It helps smooth fluctuations in **time series data**, especially in the **absence of trend or seasonality**.

$$\hat{Y}_{t+1} = \frac{Y_t + Y_{t-1} + \cdots + Y_{t-n+1}}{n}$$

- n = number of periods
- $\hat{Y}_{t+1}$ = forecast for the next period

The choice of number of periods will affect the forecasting results.

- **Smaller n:** more sensitive to changes (less smoothing)
- **Larger n:** More smoothing, but slower to react to shifts

Figure 2. Stock Trading Using Moving Average

3 MOVING AVERAGE MODEL – EXAMPLE

Given the historical demand from Period 1 to 10, use 3-period moving average to forecast the demand in Period 4 to Period 11. Plot the observed demand vs 3-period MA.

Period	Demand (Y)	3-MA
1	120	
2	135	
3	150	
4	140	135
5	170	141.67
6	175	153.33
7	165	161.67
8	185	
9	170	
10	200	
11		

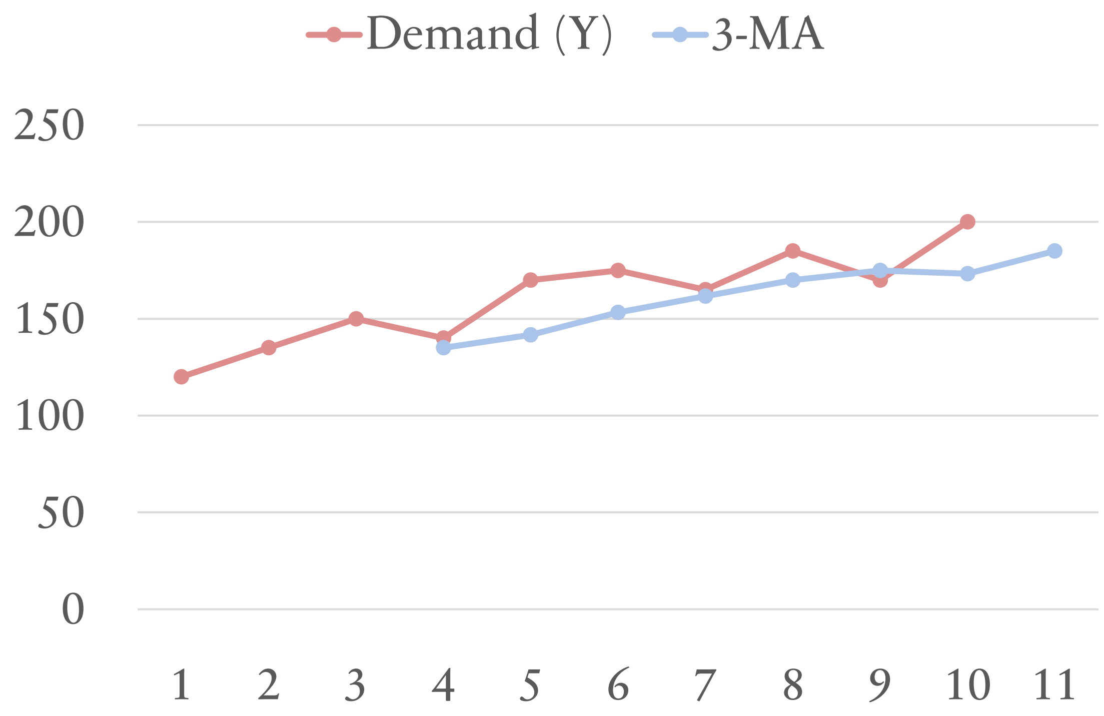
$$\hat{y}_4 = \frac{120 + 135 + 150}{3} = 135$$

$$\hat{y}_5 = \frac{135 + 150 + 140}{3} = 141.67$$

$$\hat{y}_6 = \frac{150 + 140 + 170}{3} = 153.33$$

$$\hat{y}_7 = \frac{140 + 170 + 175}{3} = 161.67$$

3 MOVING AVERAGE MODEL – EXAMPLE



- MA smooths out spikes and noise in the data
- Best fit:
  - Stable demand patterns
  - Short-term forecasting

Limitations

- Lagging indicator – always behind actual data
- Cannot detect trend or seasonality
- Weights are all equal – all recent data treated the same (average)

3 MOVING AVERAGE MODEL – EXERCISE

A warehouse records the weekly demand (in units) for a specific component. Use moving averages with different periods to forecast demand and evaluate their effectiveness.

Week	1	2	3	4	5	6	7	8	9
Demand	200	220	210	240	230	250	270	260	240

Instructions:

1. Compute forecasts using:

(i) 2-period moving average

(ii) 3-period moving average

(iii) 4-period moving average

(Start calculating forecasts only when you have enough prior data points.)

2. Compute the SSE for each model.

3. Which moving average performed best?

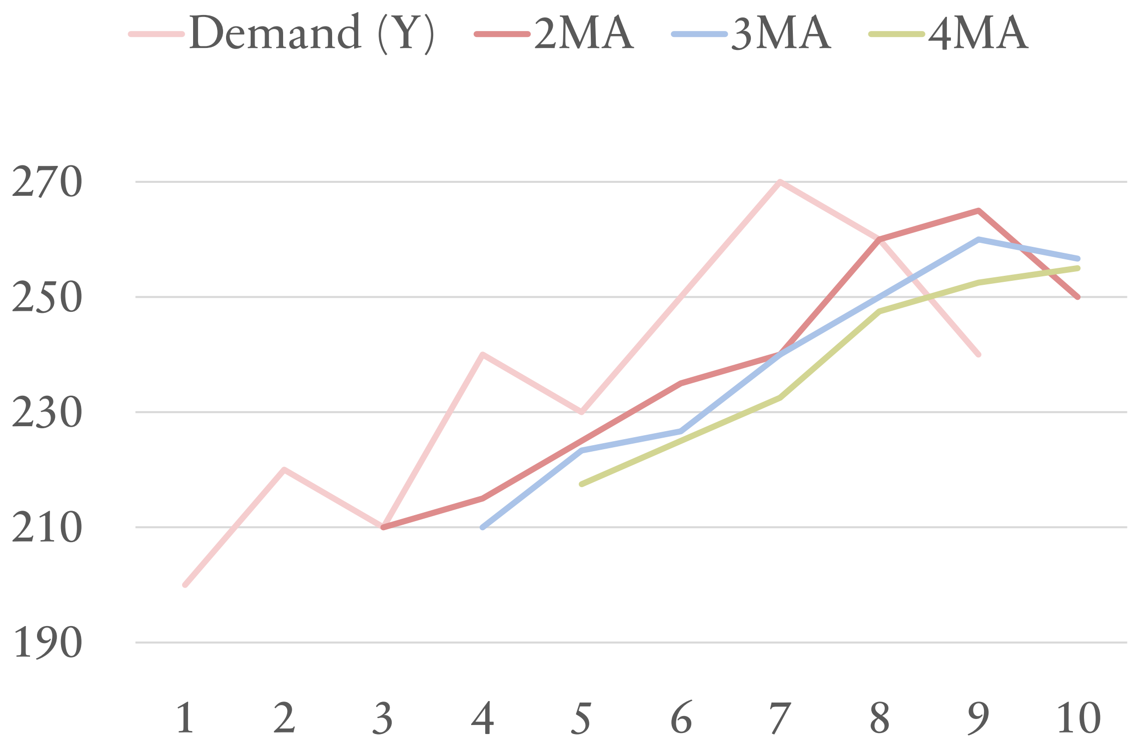
4. How does increasing n affect the responsiveness of the model?

5. Which model would you choose if demand becomes more volatile?

3 MOVING AVERAGE MODEL – EXERCISE

1. **Compute forecasts using 2-MA, 3-MA, and 4-MA**

Period	Y	2-MA	3-MA	4-MA
1	200			
2	220			
3	210			
4	240			
5	230			
6	250			
7	270	240		
8	260	260	250	
9	240	265	260	252.5
10		250	256.67	255



3 MOVING AVERAGE MODEL – EXERCISE

2-3. Compute the SSE for each model. Which moving average performed best?

Period	Y	2-MA	3-MA	4-MA
1	200			
2	220			
3	210			
4	240			
5	230			
6	250			
7	270	240		
8	260	260	250	
9	240	265	260	252.5

4 WEIGHTED MOVING AVERAGE MODEL

- A refinement of the simple moving average.
- **Assigns more importance** (**weight**) to recent observations.
- Better for data with **slight trends**, where **recent periods are more relevant**.

$$\hat{Y}_{t+1} = w_1 Y_t + w_2 Y_{t-1} + \cdots + w_n Y_{t-n+1}$$

  - $\sum w_i = 1$
  - Recent values typically have higher weights

The choice of weights will affect the forecasting results.

- **Manual tuning:** Try weights like (0.5, 0.3, 0.2) or (0.6, 0.3, 0.1)
- **Optimisation:** Use historical error metrics to minimise forecast error
- **Heuristic:** Emphasize recency, but balance for smoothing

4 WEIGHTED MOVING AVERAGE - EXAMPLE

A logistics manager tracks weekly outbound shipment volumes for a distribution center. To better anticipate outbound volume and optimize truck scheduling, you'll forecast demand using both 3-period SMA and 3-period WMA with custom weights as follows:

- Most recent: 0.5
- Second recent: 0.3
- Third recent 0.2

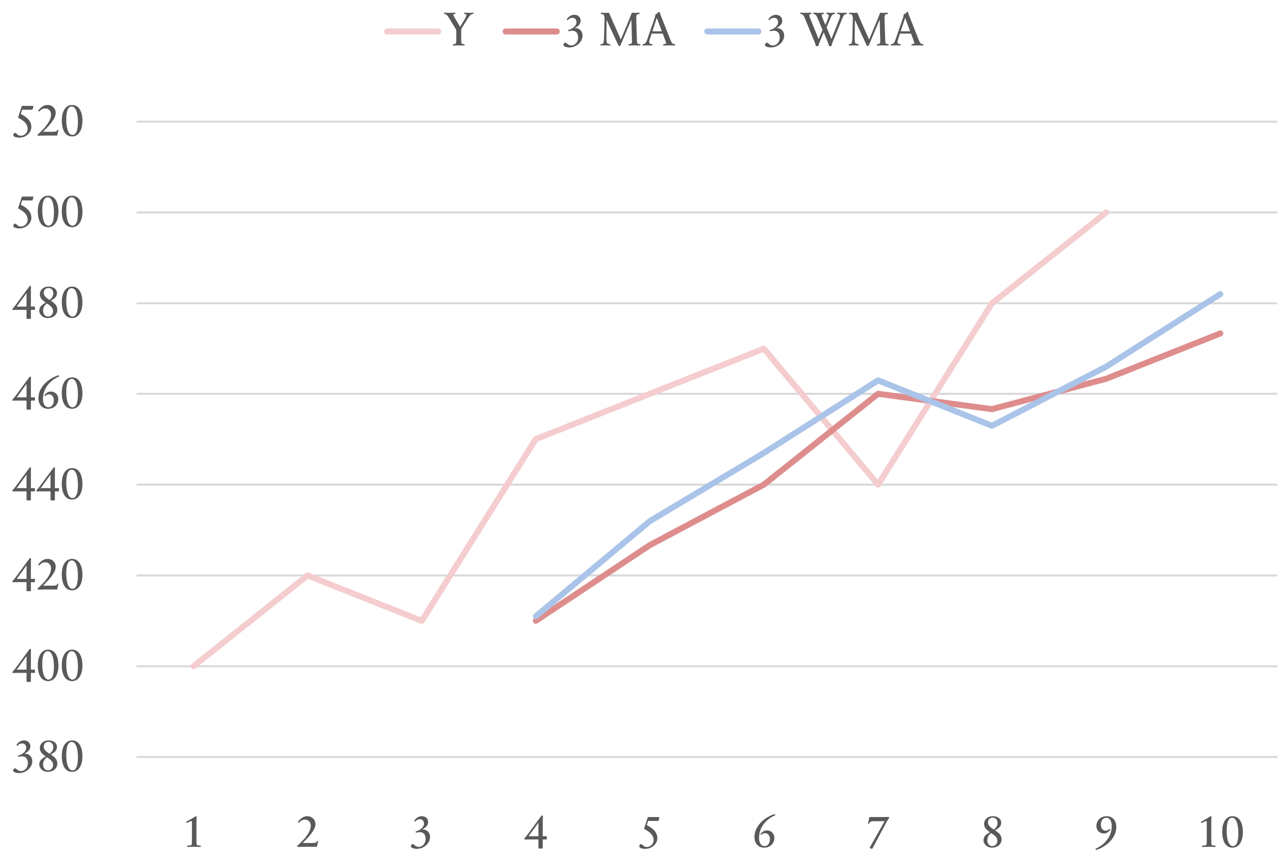
Week	1	2	3	4	5	6	7	8	9
Demand	400	420	410	450	460	470	440	480	500

4 WEIGHTED MOVING AVERAGE - EXAMPLE

X	Y	3 MA	Error (SMA)	3 WMA	Error (WMA)
1	400				
2	420				
3	410				
4	450	410.00	40.00		39
5	460	426.67	33.33		28
6	470	440.00	30.00		23
7	440	460.00	-20.00		-23
8	480	456.67	23.33		27
9	500	463.33	36.67		34
10		473.33			

Which method performed better overall? Why?

4 WEIGHTED MOVING AVERAGE - EXAMPLE



Which method performed better overall? Why?

5 SIMPLE EXPONENTIAL SMOOTHING (SES)

- SES generates forecasts by assigning exponentially decreasing weights to past observations.
- Unlike moving averages, SES automatically adjusts the weight of new data through a smoothing parameter α.

$$\hat{Y}_{t+1} = \alpha Y_t + (1 - \alpha)\hat{Y}_t$$

- α = smoothing constant $(0 < \alpha \leq 1)$
- Y_t = actual value at time t
- $\hat{Y}_t$ = forecast for time t

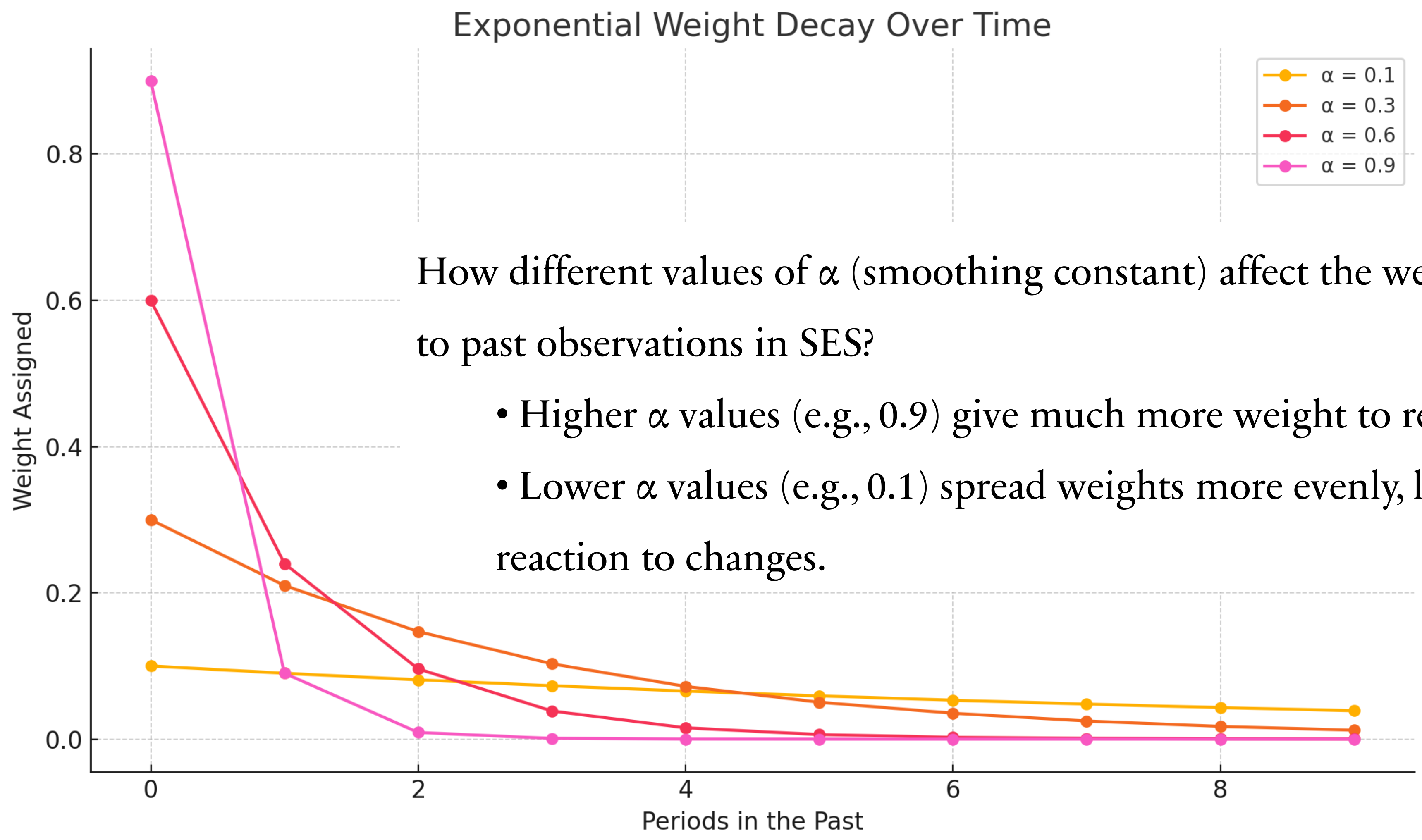
Best Fit:

- Short-term demand with no clear trend or seasonality
- Daily/weekly demand planning for consumables
- Safety stock estimation in retail or healthcare

Limitations:

- Doesn't handle trends or seasonal cycles
- Requires good choice of α
- Initial forecast selection can affect accuracy
- Forecasts are always lagging

5 SIMPLE EXPONENTIAL SMOOTHING (SES)



How different values of α (smoothing constant) affect the weights assigned to past observations in SES?

- Higher α values (e.g., 0.9) give much more weight to recent data.
- Lower α values (e.g., 0.1) spread weights more evenly, leading to slower reaction to changes.

5 SIMPLE EXPONENTIAL SMOOTHING (SES)

Tuning the Smoothing Parameter

α Value	Behaviour	Use Case
0.1 – 0.3	Slow, more smoothing	Stable demand
0.4 – 0.7	Balanced response	Mild variability
0.8 – 1.0	Fast, less smoothing	Erratic or fast-changing demand

Tip: Use MAE or RMSE to pick optimal α using trial/error or optimisation.

5 SES – EXAMPLE

A supply chain manager wants to forecast weekly demand using Simple Exponential Smoothing (SES).

- Smoothing constant: $\alpha = 0.3$
- Initial forecast $\hat{Y}_1 = 200$

$$\hat{Y}_{t+1} = \alpha Y_t + (1 - \alpha)\hat{Y}_t$$

Period	Actual Demand	Forecast	Calculation
1	200	200 (given)	Initial forecast
2	220	200	$0.3 \times 200 + 0.7 \times 200$
3	210	206	$0.3 \times 220 + 0.7 \times 200$
4	230	207.20	$0.3 \times 210 + 0.7 \times 206$
5	225	214.04	$0.3 \times 230 + 0.7 \times 207.2$
6	240	217.33	$0.3 \times 225 + 0.7 \times 214.04$
7		224.13	$0.3 \times 240 + 0.7 \times 217.33$

6 COMPARATIVE INSIGHTS

Summary of Smoothing Techniques

Method	Reactivity	Suitable For
Simple Moving Average	Low	Stable, low noise
Weighted Moving Average	Medium	Mild trend, prioritize recency
Exponential Smoothing	Variable	Short-term, adaptive

7 PRACTICE QUESTION

You are the inventory analyst for a distribution center that tracks weekly product demand. You are tasked with using four different forecasting methods to predict demand for the upcoming week and evaluate their performance.

Week	1	2	3	4	5	6	7
Demand (Units)	120	130	125	135	140	150	145

(i) Treat Week as the independent variable X and Demand as the dependent variable Y.
(ii) Calculate the regression equation using the least squares method.
(iii) Use the equation to forecast demand for Week 8.
(iv) Using a **3-period SMA**, compute the forecast for **Week 8**.
(v) Using a **3-period WMA** with weights: 0.5, 0.3, and 0.2 for most recent, middle, and oldest. Compute the forecast for Week 8.
(vi) Using SES with smoothing constant 0.4 and initial forecast 120. Compute the forecast for Week 8.
(vii) Compute the **forecast errors** (**absolute error**) for Weeks 4–7 using SMA, WMA, and SES.
(viii) Which method would you recommend for short-term forecasting in this case, and why?