SUPPLY CHAIN MODELLING AND OPTIMIZATION

TOPIC 6 INVENTORY MODEL

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1. INTRODUCTION TO INVENTORY

Inventory — The stock of any item or resource used in an organisation.

Inventory system — Policies and controls to monitor stock levels and reorder timing/quantity.

Types of Inventory

Direct

Inventory that directly contributes to the production of finished goods. These are physical items that are **integrated into the final product** or play an active role in the manufacturing process.

E.g. Raw materials, WIP (work-in-process), finished goods, spare parts.

Indirect

Inventory that does not become part of the final product but is necessary to support the production or business operations.

Fluctuation	Buffer stock, e.g. extra paper kept nearby a printer
Anticipation	Stock built ahead, e.g. fans stocked before summer
Decoupling	Safety stock that is set aside, e.g. extra raw materials
Transportation	Items in transit

1.1 INVENTORY METRICS

Average aggregate Inventory value

The total average value of inventory held across all items, which helps track the financial investment tied up in inventory.

$$\sum_{\text{all items}} (\text{Average Units} \times \text{Cost per Unit})$$

Weeks of Supply

How long the average inventory can support demand.

$$\frac{\text{Average aggregate inventory value}}{\text{Average Weekly Cost of Goods Sold (COGS)}}$$

- A high value may indicate overstocking or slow-moving inventory.
- A low value may suggest lean inventory or potential stockouts.

Inventory Turnover

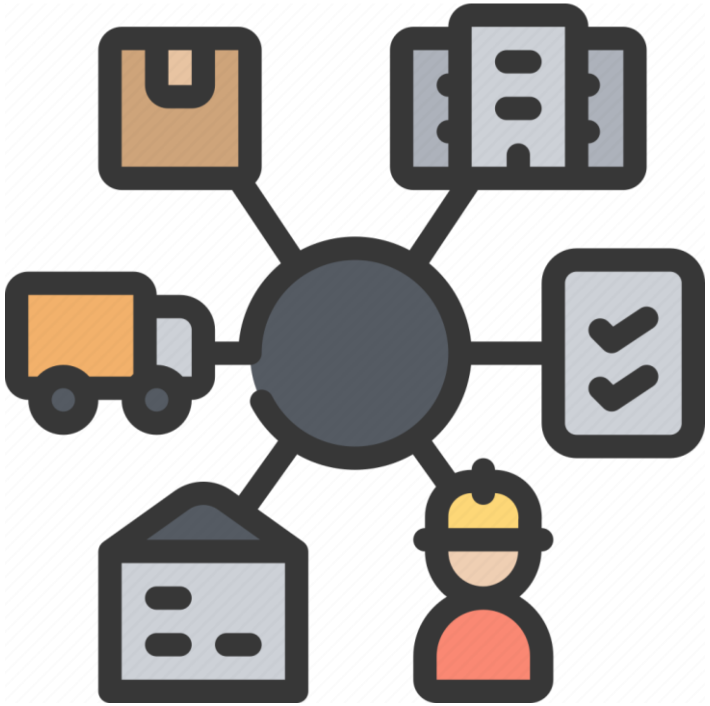
The number of times inventory is sold and replaced in a year, indicating how efficiently inventory is being used.

$$\frac{\text{Average aggregate inventory value}}{\text{the value of the sales per week}}$$

- A higher turnover implies faster sales or better inventory control.
- A lower turnover could signal overstocking, obsolete items, or slow sales.

1.2 INVENTORY CONTROL OBJECTIVES

- To satisfy expected demand.
- To protect against stock outs
- To take advantage of economic order cycles.
- To maintain independence of operations.
- To allow for smooth and flexible production operations.
- To guard against price increases



Decide → **When** to order?

Decide → **How much** to order?

The quantity ordered is called **the economic order quantity** (**EOQ**)

1.3 INVENTORY COST COMPONENTS

Component	Description
Purchase Cost (PC)	The amount paid to acquire inventory.
Inventory Holding Cost (IHC)	The cost of storing and maintaining inventory over time, including storage, insurance, depreciation, capital.
Shortage Cost (SC)	The cost incurred when demand cannot be met due to stockouts, which includes lost sales and emergency orders.
Ordering Cost (OC)	The cost associated with placing an order or setting up a production batch.
Total Inventory Cost (TC)	With discount: TC = **PC** + IHC + SC + OC Without discount: TC = IHC + SC + OC

Objective: Determine the correct quantity to order that minimise the total cost.

1.4 OTHER VARIABLES

Apart from costs, the other variables which play an important role in decision making are as follows:

Demand

- **Deterministic:** The exact quantity and timing are predictable and do **not vary** over time.
- **Probabilistic:** The demand is uncertain and described using probability distributions.

Lead-time

- **Deterministic:** One needs to order in advance by a time equal to lead-time.
- **Probabilistic:** Uncertain, so it is very difficult to decide when to order.

Cycle-time (T) Time between placements of two orders.

Planning Horizon The period over which a particular inventory level will be maintained.

1.5 TYPES OF INVENTORY MODELS

Deterministic Inventory Models

Based on the assumption that **all parameters and variables are known** and that there is **no uncertainty** associated with demand and replenishment of inventory stock.

Under this model, inventory is built up at a constant rate.

Stochastic Inventory Models

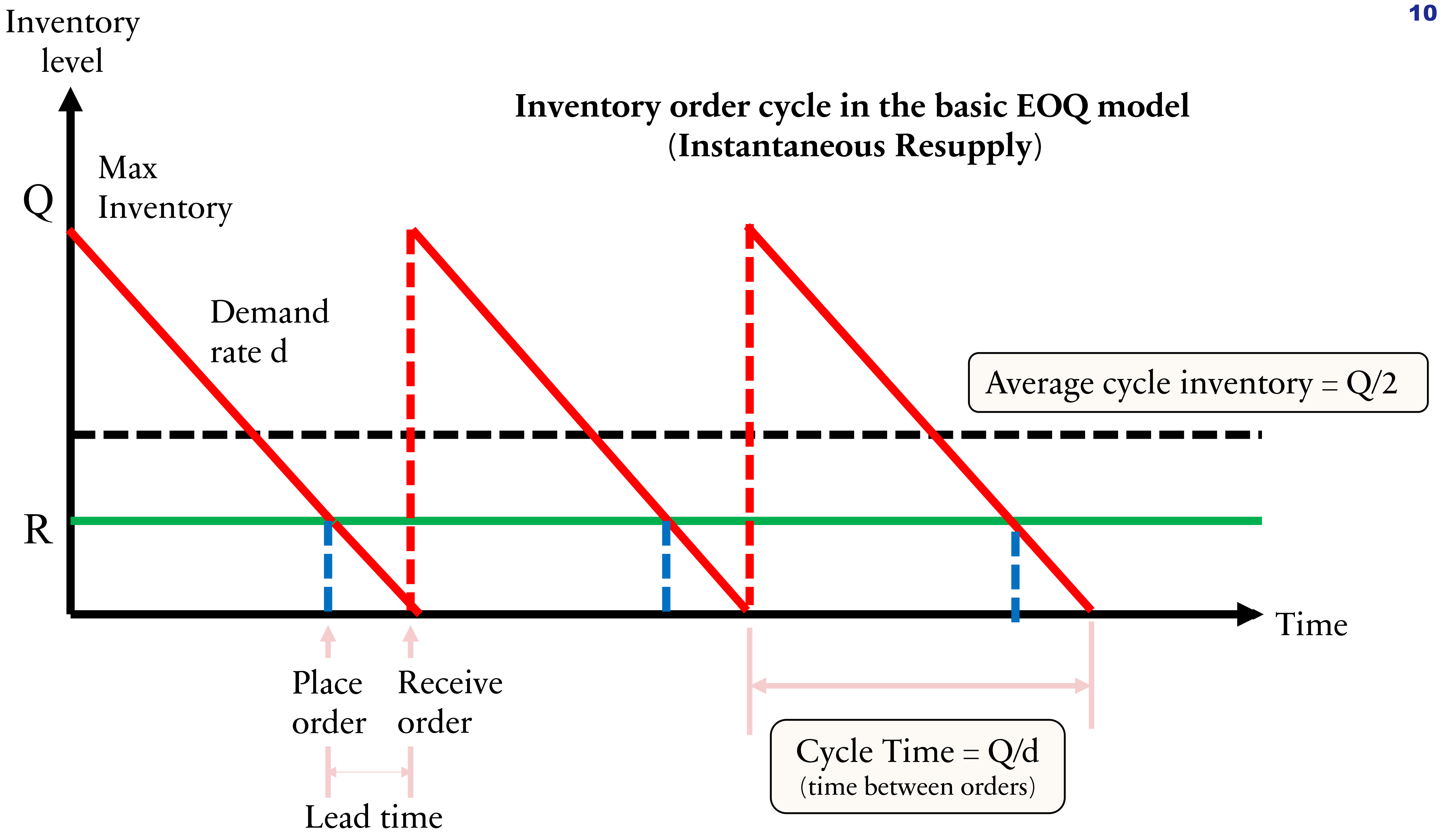
In these models, the demand can change unexpectedly and it is more important to get systems to track the inventory level.

2 DETERMINISTIC MODEL – EOQ MODEL

Assumes constant, known demand & lead times

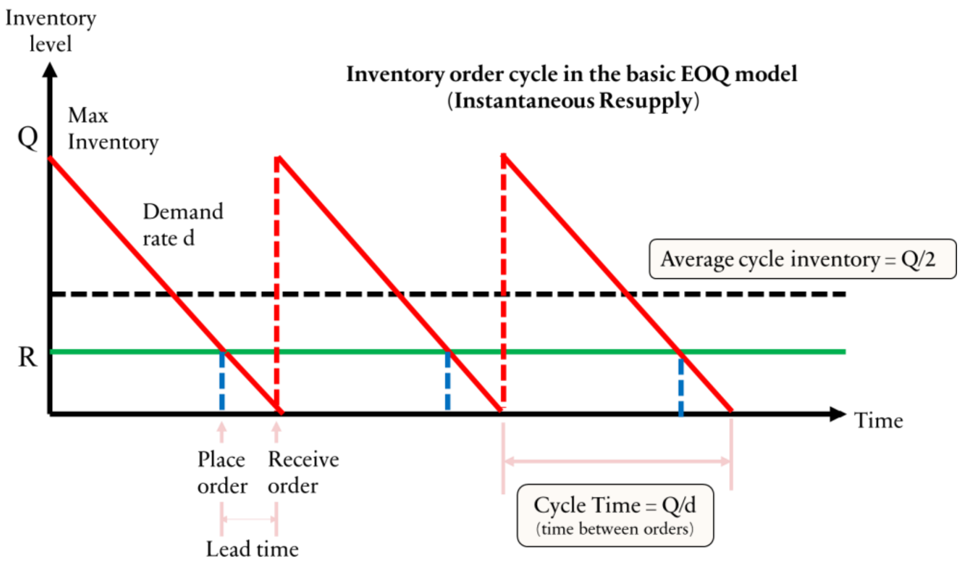
Notation	Description
C	Purchase or manufacturing cost of an item
A	The ordering cost per order.
h	Inventory holding cost of an item per unit per time unit.
k	Shortage cost per unit short per time.
d	Demand rate of an item per planning horizon (usually per year).
Q	Order quantity (a decision variable).
T	Cycle time (a decision variable).
R	Reorder level i.e. the level of inventory at which an order is placed.
L	Lead time.

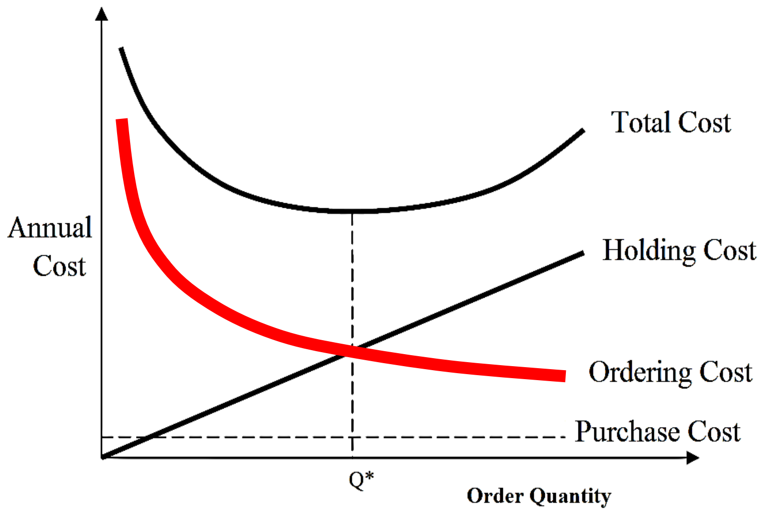
Inventory order cycle in the basic EOQ model (Instantaneous Resupply)



Burnham and Mohanty (1988) advocate for a Uniform Order Quantity.

2.1 EOQ MODEL – FORMULA





Inventory Holding Cost

$$IHC = \left(\frac{Q}{2}\right) \cdot h$$

Average cycle inventory multiply holding cost per unit (h)

Ordering Cost

$$OC = \left(\frac{d}{Q}\right) \cdot A$$

d/Q the number of orders placed during planning horizon
A the ordering cost per order

Total Inventory Cost

$$TC = \left(\frac{d}{Q}\right) \cdot A + \left(\frac{Q}{2}\right) \cdot h$$

= OC + IHC

2.1 EOQ MODEL – FORMULA

Goal: Find the order quantity Q^* that **minimise total cost**

Total Inventory Cost

$$TC = \left(\frac{d}{Q}\right) \cdot A + \left(\frac{Q}{2}\right) \cdot h$$

Setting the first derivative of TC w.r.t. Q to zero and solving for Q:

$$\frac{d}{dQ} TC = \frac{-d}{Q^2} A + \frac{h}{2} = 0$$

Optimal lot size

$$Q^* = \sqrt{\frac{2dA}{h}}$$

The minimum TC

$$TC(Q^*) = \left(\frac{d}{Q^*}\right) \cdot A + \left(\frac{Q^*}{2}\right) \cdot h$$

$$TC(Q^*) = \sqrt{2dAh}$$

If **quantity discount** is taken into account, then $TC = \boldsymbol{CD} + \left(\frac{d}{Q^*}\right) \cdot A + \left(\frac{Q^*}{2}\right) \cdot h$

2.1 EOQ MODEL – FORMULA

How often should an order be placed to maintain inventory efficiently?

Cycle Time	$T = \frac{Q^*}{d}$	The time interval **between placing two consecutive orders** (in years, months, or days depending on units).
Order Frequency	$\frac{d}{Q^*}$	Number of orders **placed** per planning horizon (in years, months, or days depending on units).

These two are *reciprocals* of each other.

2.2 SENSITIVITY OF THE LOT-SIZE MODEL

- The sensitivity analysis of the lot-size model examines how changes in the order quantity (from the optimal EOQ value) affect the total cost.
- It helps determine how critical it is to order exactly the Economic Order Quantity (EOQ), or whether near-optimal values are acceptable.

Why is it important?

- In real-world situations, you might not always order exactly EOQ due to supplier constraints, quantity discounts, or packaging sizes.
- Sensitivity analysis shows that the total cost curve is relatively flat near EOQ, meaning minor deviations in order size result in minimal cost changes.

2.2 EOQ MODEL

Sensitivity of the Lot-Size Model (Continue)

If Q^* = EOQ (optimal lot size)

$Q_1 = \text{bQ}^*$, where $b > 0$ is a multiplier (e.g., 0.8, 1.2)

Then, **the sensitivity ratio** is

$$\frac{TC(Q_1)}{TC(Q^*)} = \frac{1 + b^2}{2b}$$

$TC(Q_1)$ is the total cost at adjusted quantity
$TC(Q^*)$ is the total cost at EOQ

Example 1. Let's say you deviate from EOQ by **20%** (i.e., b=1.2)

$$\frac{TC(Q_1)}{TC(Q^*)} = \frac{1 + 1.44}{2.4} = \frac{2.44}{2.4} \approx 1.017$$

This means your total cost increases by only 1.7% — very small!

So, if you cannot order exactly at EOQ, small changes around it **won't significantly impact your cost.**

2.3 EOQ MODEL – EXAMPLE

Example 2. Determine the optimal lot size and the total variable cost associated with the policy of ordering quantities of that size. Annual **demand of 20,000** units, **ordering cost of $150 per order**, and **annual inventory carrying cost is $0.24 per unit**.

$$Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2(20,000)(150)}{0.24}} = 5,000 \quad \text{units}$$

$$TC(Q^*) = \left(\frac{d}{Q^*}\right) \cdot A + \left(\frac{Q^*}{2}\right) \cdot h = \left(\frac{20000}{5000}\right) \cdot 150 + \left(\frac{5000}{2}\right) \cdot 0.24 = \$1,200$$

Alternatively,

$$TC(Q^*) = \sqrt{2dAh} = \sqrt{2(20000)(150)(0.24)} = \$1,200$$

2.3 EOQ MODEL – EXAMPLE

Example 3. A company uses rivets at a rate of 5,000 kg per year, rivets costing $2 per kg. It costs $20 to place an order and the carrying cost of inventory is 10% per annum. How frequently should order for rivets be placed and how much?

Given d = 5,000 kg/year.

C = $2 per kg.,

A = $20 per order

h = C × 10% = 0.2 **per unit per year.**

$$Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2(5{,}000)(20)}{0.2}} = \mathbf{1{,}000} \quad \text{kgs}$$

$$T = \frac{Q^*}{d} = \frac{\mathbf{1000}}{5000} = 0.2 \ \textbf{years} \quad = 2.4 \ \textbf{months}$$

2.3 EOQ MODEL – EXAMPLE

Example 4. A supplier ships 100 units of a product every Monday. The purchase cost of product is \$60 per unit. The cost of ordering and transportation from the supplier is \$150 per order. The cost of carrying inventory is estimated at 15 % per year of the purchase cost. Find the lot-size that will minimize the cost of the system. Also determine the optimum inventory cost.

Given d = 100 units/**week = 100(52) = 5200 units/year**

C = \$60 per unit

A = \$150 per order

h = C × 15% = \$9 **per unit per year.**

$$Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2(5{,}200)(150)}{9}} = 416.3333$$

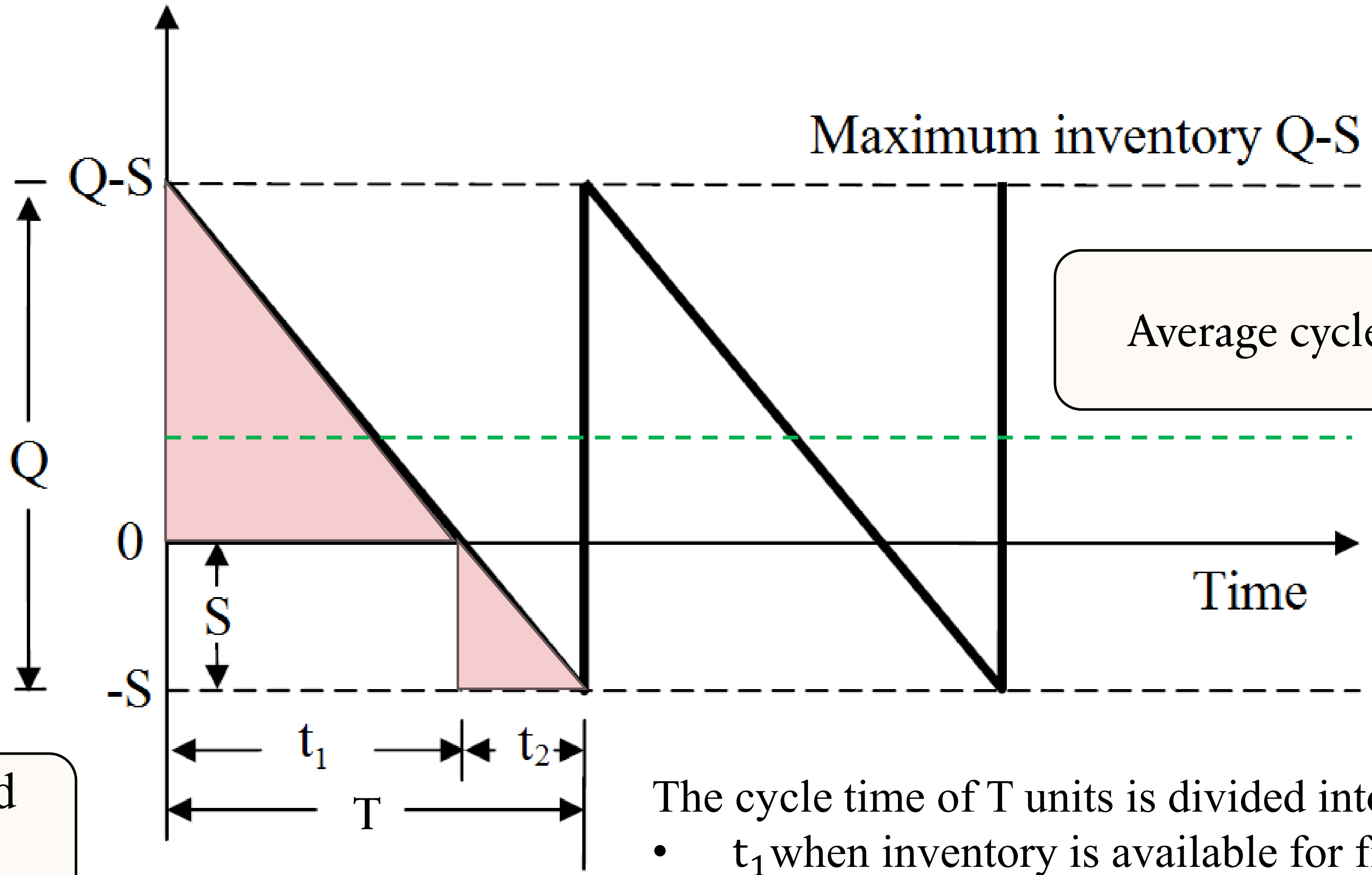
Round up: Slightly more inventory, minimal cost increase, but lower risk of shortage ≈ 417

Round down: Preferred if strict space/budget, lower holding costs, but riskier to stockouts. ≈ 416

$$TC(Q^*) = \left(\frac{5200}{416}\right) \cdot 150 + \left(\frac{5200}{2}\right) \cdot 9 = \$25{,}275$$

New variables: Shortage cost (k) and Shortage quantity (S)

2.4 EOQ MODEL WITH SHORTAGE ALLOWED



$$\text{Average cycle inventory} = \left(\frac{Q-S}{2}\right)$$

$t_1 = (Q - S) / d$

$t_2 = S / d$

The cycle time of T units is divided into:

- t_1 when inventory is available for filling orders and
- t_2 when inventory is not available, stock outs occur, and back orders are made.

2.4 EOQ MODEL WITH SHORTAGE ALLOWED

Inventory Holding Cost

$$IHC = \frac{(Q-S)^2}{2Q} \cdot h$$

Shortage Cost

$$SC = \left(\frac{S^2}{2Q}\right) \cdot k$$

Average number of units short multiply shortage cost per unit

Total Inventory Cost

= OC + IHC + SC

$$TC = \frac{d}{Q^*} \cdot A + \frac{(Q-S)^2}{2Q^*} \cdot h + \frac{S^2}{2Q^*} \cdot k$$

$$TC^* = \sqrt{2dAh\left(\frac{k}{k+h}\right)}$$

Setting the first derivative of TC w.r.t. Q and S to zero and solving for Q and S,

Optimal lot size

$$Q^* = \sqrt{\frac{2dA}{h}\left(\frac{k+h}{k}\right)}$$

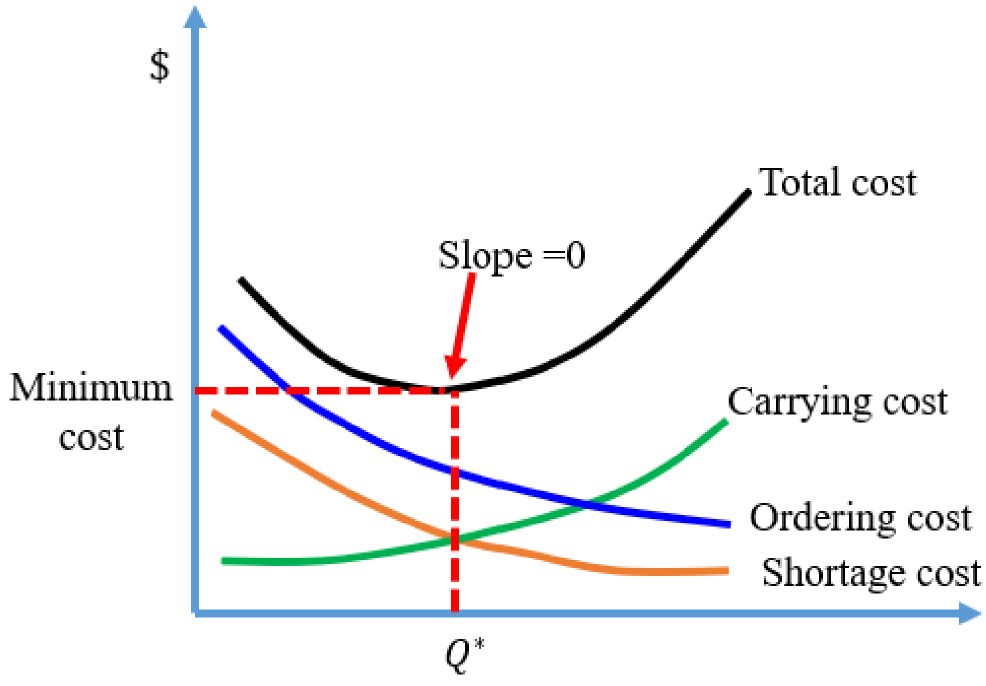
Optimal shortage size

$$S^* = Q^*\left(\frac{h}{k+h}\right)$$

2.4 EOQ MODEL WITH SHORTAGE ALLOWED

Other formulas

- ✓ The number of orders for the planning horizon = d/Q*
- ✓ The maximum inventory level = Q* - S*
- ✓ The average inventory level is = (Q* - S*) / 2
- ✓ The length of the inventory cycle = T* = Q* / d



2.4 EOQ MODEL WITH SHORTAGE ALLOWED

Example 5. The annual demand for a certain item is 3,000 units per period. Unsatisfied demand causes a shortage cost of $0.75 per unit per year. The cost of initiating purchasing action is $15.00 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is $8.00 per unit. Find purchase quantity and the minimum inventory cost.

Given

d = 3,000 units

C = $8 per unit

k = $0.75 per unit per year

h = $8 × 15% = $1.20

A = $15.00 per order.

$$Q^* = \sqrt{\frac{2dA}{h}\left(\frac{k+h}{k}\right)} = \sqrt{\frac{2(3000)(15)}{1.20}\left(\frac{0.75+1.20}{0.75}\right)} = 441.58 \approx 442$$

$$TC^* = \sqrt{2dAh\left(\frac{k}{k+h}\right)} = \sqrt{2(3000)(15)(1.20)\left(\frac{0.75}{0.75+1.20}\right)} = \$203.81$$

2.4 EOQ MODEL WITH SHORTAGE ALLOWED

Example 6. A television manufacturing company produces its own speakers, which are used in the production of its television sets. The television sets are assembled on a continuous production line at rate of 8,000 per month. The company is interested in determining when and how much to procure, given the following information:

- Each time a batch is produced, a set-up cost of $12,000 is incurred.
- The cost of keeping a speaker in stock is $ 0.30 per month.
- The production cost of a single speaker is $10.00 and can be assumed to be a unit cost.
- Shortage of a speaker, (if there exists) costs $ 1.10 per month.

Given d = 8000 units per month; A = $12,000 per batch; h = $0.3 per unit per month; k = $1.10 per month

When shortages are <u>not allowed</u>

$$Q^* = \sqrt{\frac{2dA}{h}} = \sqrt{\frac{2(8000)(12000)}{0.3}} = 25{,}298 \quad \text{speakers}$$

$$T = \frac{Q^*}{d} = \frac{25298}{8000} = 3.16 \textbf{ month}$$

∴ 25,298 speakers are to be produced every 3.2 months.

2.4 EOQ MODEL WITH SHORTAGE ALLOWED

Given $d = 8000$ units per month $\quad h = \$0.3$ per unit per month

$A = \$12{,}000$ per batch $\quad k = \$1.10$ per month

When shortages are allowed

$$Q^* = \sqrt{\frac{2dA}{h}\left(\frac{k+h}{k}\right)} = \sqrt{\frac{2(8000)(12000)}{0.3}\left(\frac{1.10+0.3}{1.10}\right)}$$

$$= 28540 \quad \text{speakers}$$

$$T = \frac{Q^*}{d} = \frac{28540}{8000} = 3.57 \textbf{ month}$$

∴ 28,540 speakers are to be produced every 3.6 months.

Optimal number of speakers stored $= Q^*\left(\frac{k}{k+h}\right) = 22{,}424$ speakers

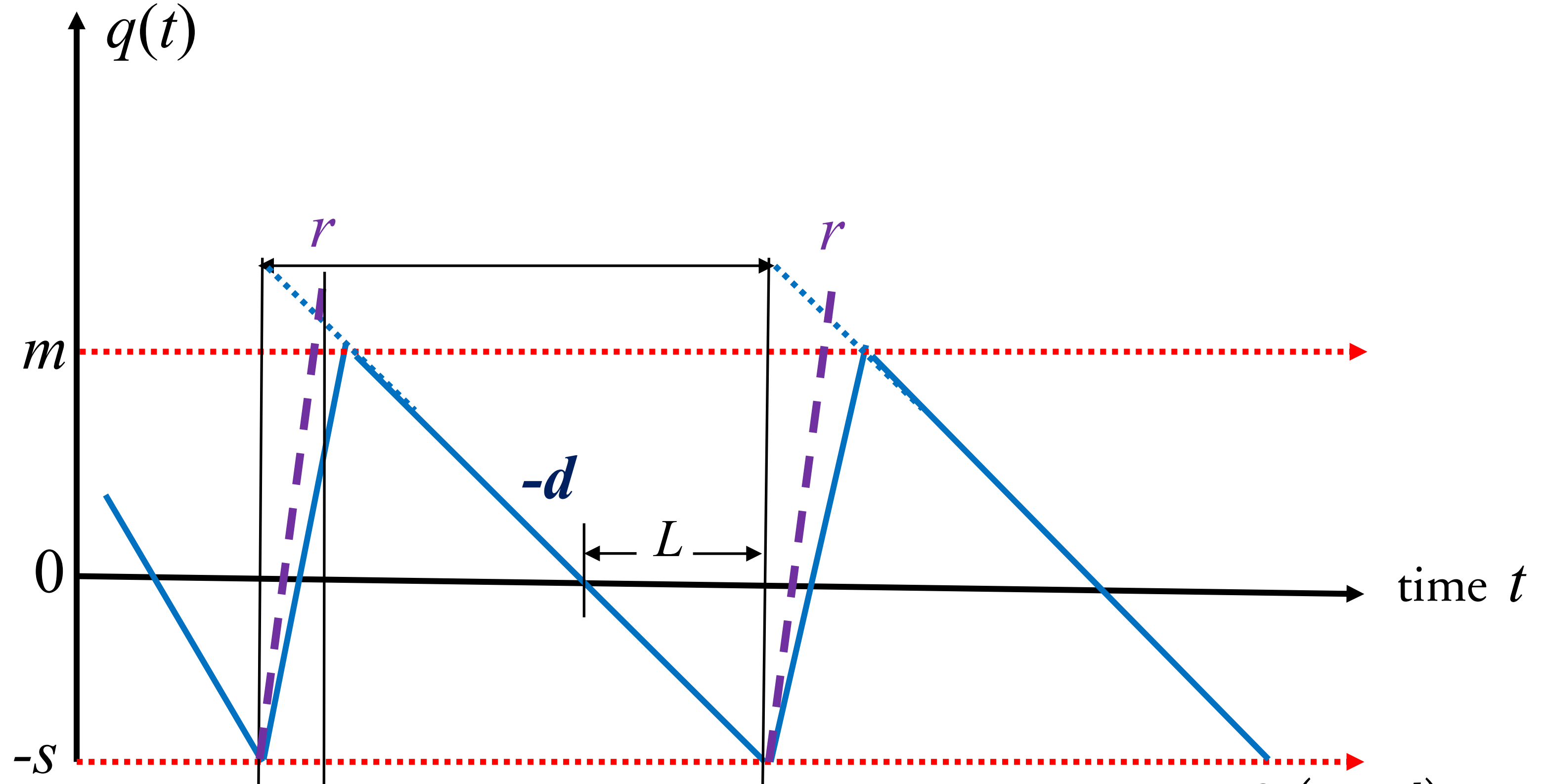
A shortage of 6,116 (= 28,540 – 22,424) speakers is permitted.

The optimal shortage level $= S^* = Q^*\left(\frac{h}{k+h}\right) \approx 6{,}116$

Replenishment = restoration of stock or supply to the former level

3 ECONOMIC PRODUCTION QUALITY (EPQ) MODEL

For production environments with production rate ***r*** and usage rate ***d***



Replenishment time $T_r = Q/r$

$T = Q/d = T_r + T_d$

Depletion time $T_d = \frac{Q}{d}\left(1 - \frac{d}{r}\right)$

3 ECONOMIC PRODUCTION QUALITY (EPQ) MODEL

For production environments with production rate ***r*** and usage rate ***d***

Inventory Holding Cost

$$IHC = \left(1 - \frac{d}{r}\right)\frac{Q}{2}h$$

Total Inventory Cost

= OC + IHC

$$TC = \frac{d}{Q} \cdot A + \left(1 - \frac{d}{r}\right)\frac{Q}{2}h$$

$$TC^* = \sqrt{2dAh\left(1 - \frac{d}{r}\right)}$$

Setting the first derivative of TC w.r.t. Q to zero and solving for Q,

Optimal lot size

$$Q^* = \sqrt{\frac{2dA}{h}\left(\frac{r}{r-d}\right)} = \sqrt{\frac{2dA}{h(1-d/r)}}$$

3 ECONOMIC PRODUCTION QUALITY (EPQ) MODEL

Example 7. A product is to be manufactured on a machine. The cost, production, demand, etc. are as follows:

- Ordering cost per order = \$30;
- Purchase cost per unit = \$0.10
- Inventory holding cost per unit per annum = \$0.05
- Production rate = 100,000 units per year
- Demand rate = 10,000 units / year.

Determine the economic manufacturing quantity.

Given A= \$30 per order,

C = \$0.10 per unit,

h = 0.05 / unit / annum,

r = 100,000 units per year and

d = 10,000 units per year

$$Q^* = \sqrt{\frac{2dA}{h}\left(\frac{r}{r-d}\right)}$$

$$= \sqrt{\frac{2(10{,}000)(30)}{0.05}\left(\frac{100{,}000}{90{,}000}\right)} = 3651 \quad \text{units}$$

Unit must always be CONSISTENT.

3 ECONOMIC PRODUCTION QUALITY (EPQ) MODEL

Example 8. Golden Food distributes tinned foodstuff in Great Britain. In a warehouse located in Birmingham,

- The demand rate (d) for tomato purée is 400 pallets a month.
- The value of a pallet is $C = 2500$ pounds and the annual interest rate I is 14.5% (including warehousing costs).
- Issuing an order costs (A) 30 pounds.
- Number of working days in a month equals 20.
- The replenishment rate (r) is 40 pallets per day.
- Shortages are not allowed.
- The holding cost is given by $h = 0.145 \times 2500 = 362.5$ pounds **per year** per pallet

$= 30.2$ pounds **per month** per pallet

$$Q^* = \sqrt{\frac{2Ad}{h\left(1 - \frac{d}{r}\right)}} = \sqrt{\frac{2 \times 30 \times 400}{30.2\left[1 - \frac{400}{40 \times 20}\right]}} = 39.9 \approx 40 \text{ pallets}$$

$$T^* = \frac{Q^*}{d} = \frac{40}{400} = \frac{1}{10} \text{ month} = 2 \text{ workdays,} \qquad T_r^* = \frac{Q^*}{r} = \frac{40}{40} = 1 \text{ workday.}$$

4 REORDER POINT & POLICY

When to order? Order Q units when stock drops to R?

i. The **basic reorder level** (R) is calculated as

$$R = d \times L$$

$d \times L$: Demand during total lead time.

This is used when:

- Lead time (L) is shorter than or equal to the cycle time T
- There is only one order in progress at any time

ii. **Adjusted Reorder Point** (R) is calculated as

$$R = d \times L - n \times Q^*$$

$n \times Q^*$: demand that will be covered by previous orders already placed

This is used when:

- Lead time is longer than the cycle time(i.e., multiple orders may be "in flight" during the lead time)

The **Adjusted Reorder Point** is the actual remaining demand for which a new order is needed.

4 REORDER POINT & POLICY

Example 9
(Basic Reorder)

d = 100 units/week

L = 2 weeks

Then, R = 100 × 2 = 200 units

So, you place an order when stock falls to 200 units

Example 10
(Adjusted)

d = 100 units/week
L = 3 weeks
$Q^* = 250$
$T = Q^* / d = 250/100 = 2.5$ weeks
Lead time (3 weeks) is between $1T$ and $2T$, so $n = 1$

$$R = d \times L - n \times Q^* = 100 \times 3 - 1 \times 250 = 300 - 250 = 50 \text{ units}$$

You wait until stock falls to 50 units (because 250 units are already on the way) and then place another order for 250 units.