SUPPLY CHAIN MODELLING AND OPTIMIZATION

TOPIC 8 STOCHASTIC INVENTORY MODEL

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TOPIC 8 STOCHASTIC INVENTORY MODEL

1. Review on reorder point for the basic EOQ
2. Continuous review model (the Q system)
    - 2.1 Variable demand and constant lead time
    - 2.2 Constant demand and variable lead time
    - 2.3 Variable demand and variable lead time
3. Periodic review model (the P system)
4. Practice question

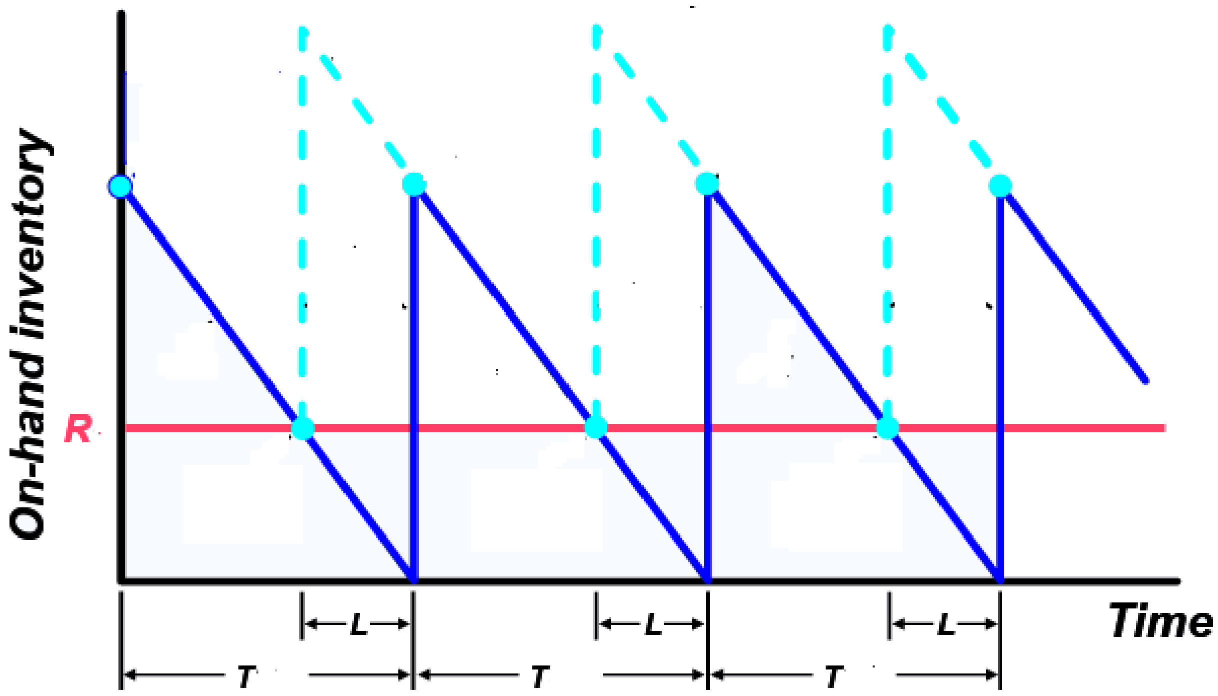
1 REORDER POINT FOR THE BASIC EOQ

The **reorder point** in EOQ model with constant demand and a constant lead time to receive an order is equal to the amount demanded during the lead time,

$$\mathbf{R = d \times L,}$$

where d = demand rate per period (i.e., daily)

L = lead time



1 REORDER POINT FOR THE BASIC EOQ

Example 1. The carpet store wants to determine the optimal order size and total inventory cost given an estimated demand of 10,000 yards of carpet, an annual carrying cost of $0.75 per yard, and an ordering cost of $150. The store would also like to know the number of orders that will be made annually and the time between orders (i.e., the order cycle) given that the store is open 311 days per year. The lead time to receive an order is 10 days, determine the reorder point for carpet.

Given D = 10,000 yards; h = $0.75 per yard per year , A = $150 , L = 10 days;

The optimal order size is

$$EOQ = Q^* = \sqrt{\frac{2DA}{h}} = \sqrt{\frac{2(10{,}000)(150)}{(0.75)}} = 2{,}000\ yards$$

The total annual inventory cost is

$$TC = \frac{D}{Q^*}A + \frac{Q^*}{2}h$$

$$= \frac{(10{,}000)(150)}{2000} + \frac{(2{,}000)(0.75)}{2}$$

$$= \$750 + \$750 = \$1{,}500$$

1 REORDER POINT FOR THE BASIC EOQ

The number of orders per year is $$\frac{D}{Q^*} = \frac{10{,}000}{2{,}000} = 5$$

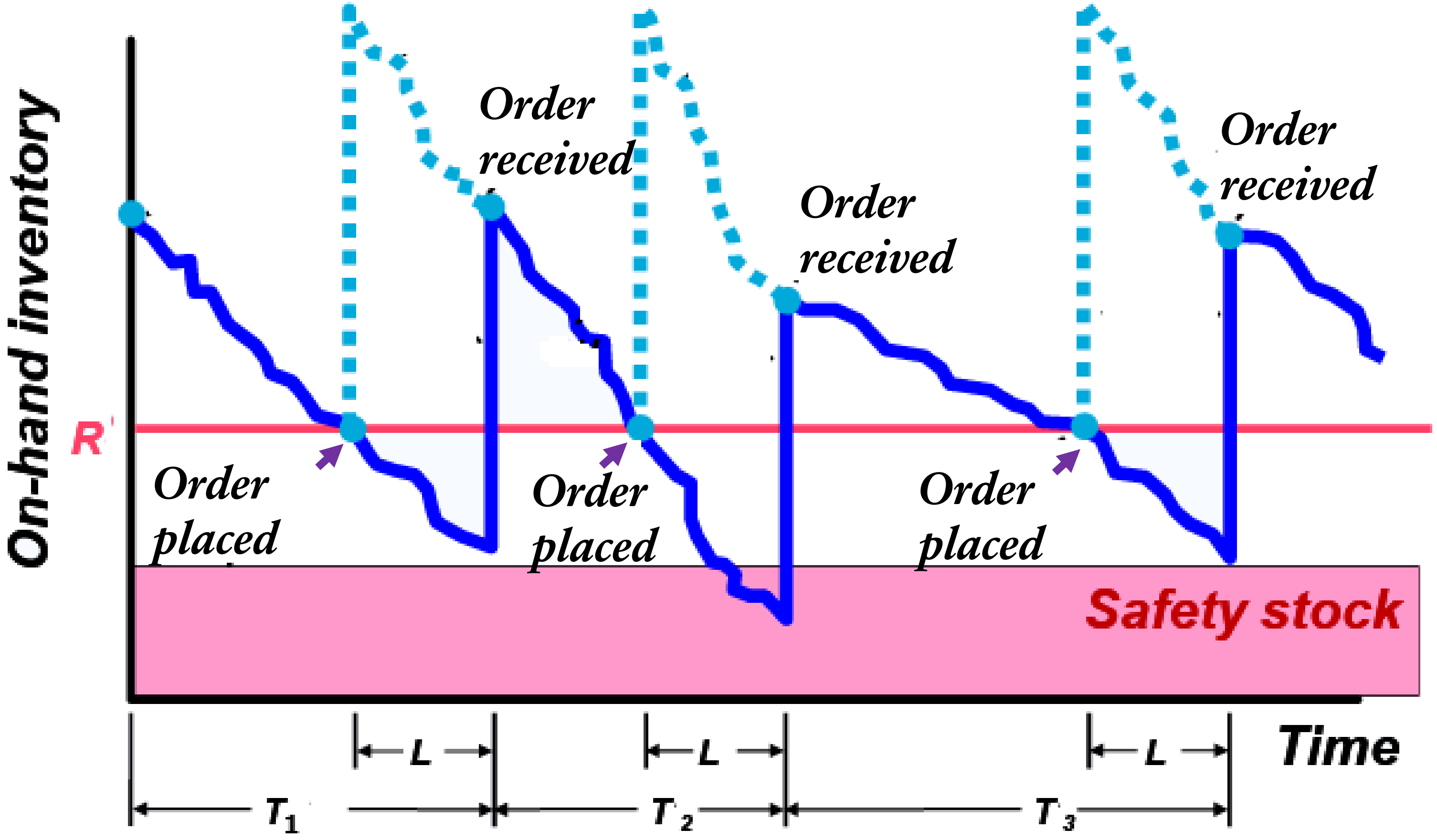
The order cycle is $$T = \frac{Q^*}{D} = \frac{1}{5} = 0.2 \text{ year} = 0.2 \times 311 = 62.2 \text{ days}$$

Reorder point is $$R = d \times L = \left(\frac{10{,}000}{311}\right)(10) = 321.54 \text{ yards}$$

But we have not considered uncertainty!

2 THE CONTINUOUS REVIEW SYSTEM (Q SYSTEM)

Uncertainty in **lead time** or **demand**, or both creates the need for safety stock (SS).



Safety stock (SS) is the extra inventory you keep on hand as a buffer to protect against uncertainties.

2.1 VARIABLE DEMAND & CONSTANT LEAD TIME

An order is placed for a variable amount after a specified period of time.

With variable demand and constant lead time,

Reorder point = Average demand during lead time + Safety Stock

$$R = \bar{d} + (z \times \sigma_{LT})$$

where $\bar{d} = d \times L$ $\sigma_{LT} = \sigma_d \sqrt{L}$

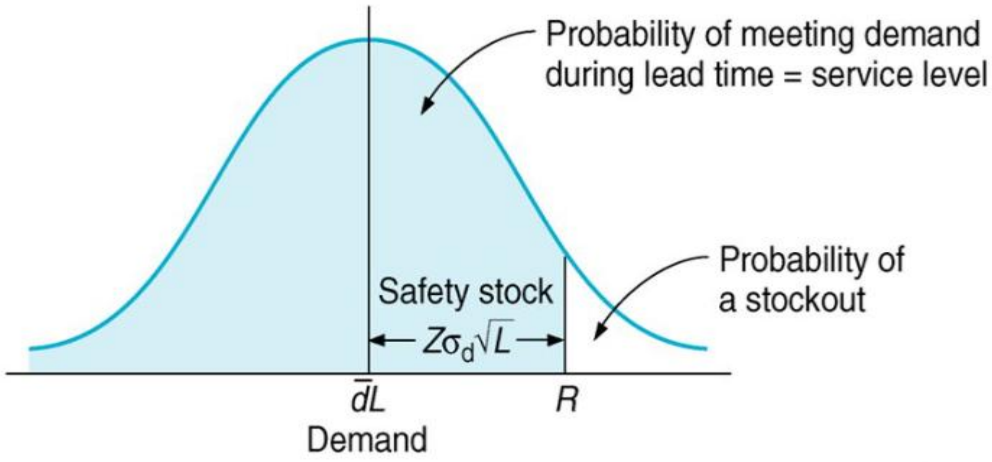
d = average demand per week (or day or month)
L = constant lead time in weeks (or days or month)
$\bar{d}$ = the average demand during lead time
z = the number of standard deviations
σ_d = the standard deviation of demand per week (or day or month)
σ_{LT} = the standard deviation of the demand during lead time

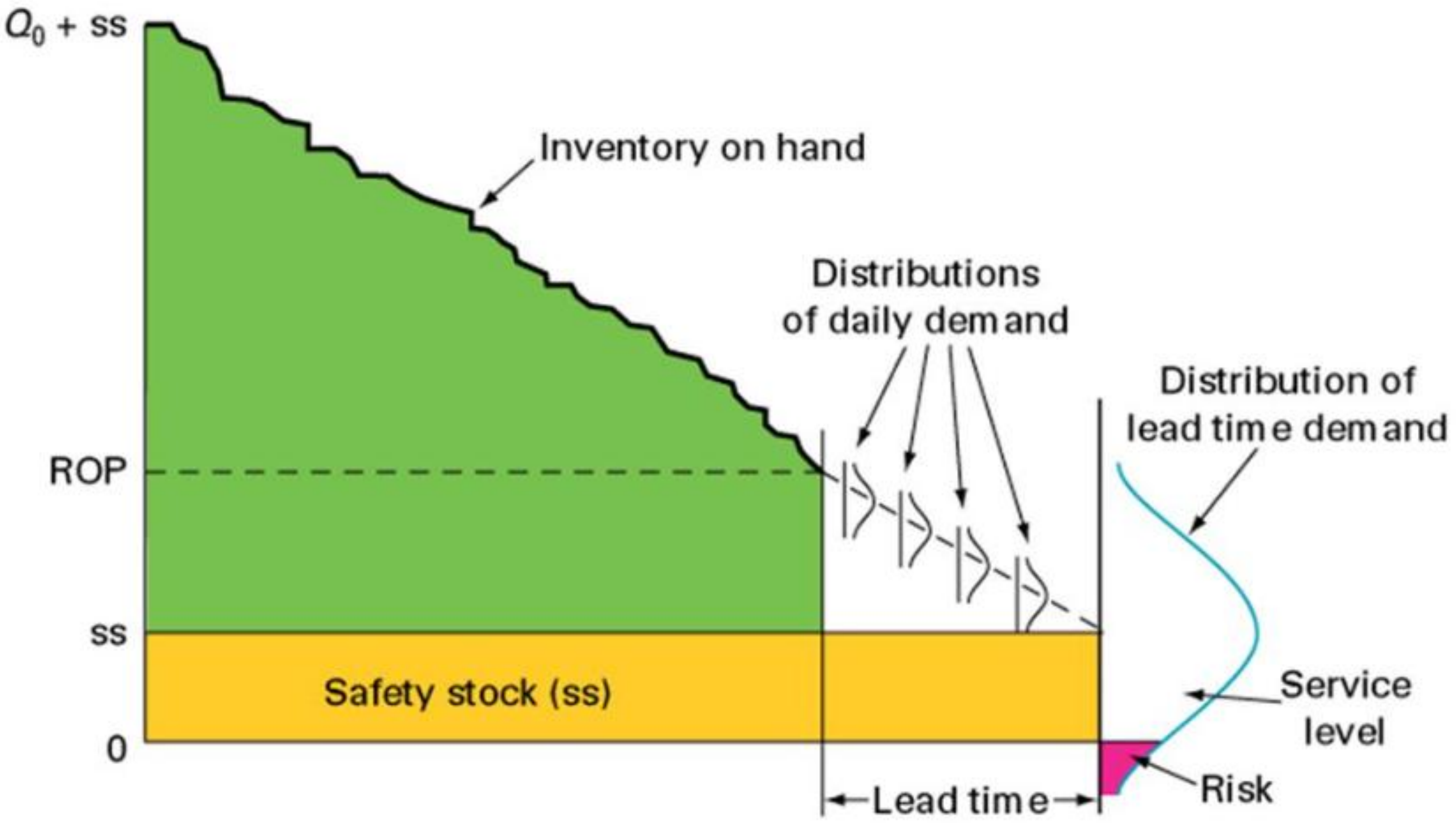
2.1 VARIABLE DEMAND & CONSTANT LEAD TIME

Service Level

One popular method is to establish a safety stock that will meet a specified **service level**, the probability that the amount of inventory on hand during the lead time is sufficient to meet expected demand – that is, the probability that a stockout will not occur.

A service level of 90% means that there is a 0.90 probability that demand will be met during the lead time, and the probability that a stockout will occur is 10%.




Q_0 + ss
Inventory on hand
Distributions of daily demand
Distribution of lead time demand
ROP
ss
Safety stock (ss)
0
Service level
Risk
Lead time

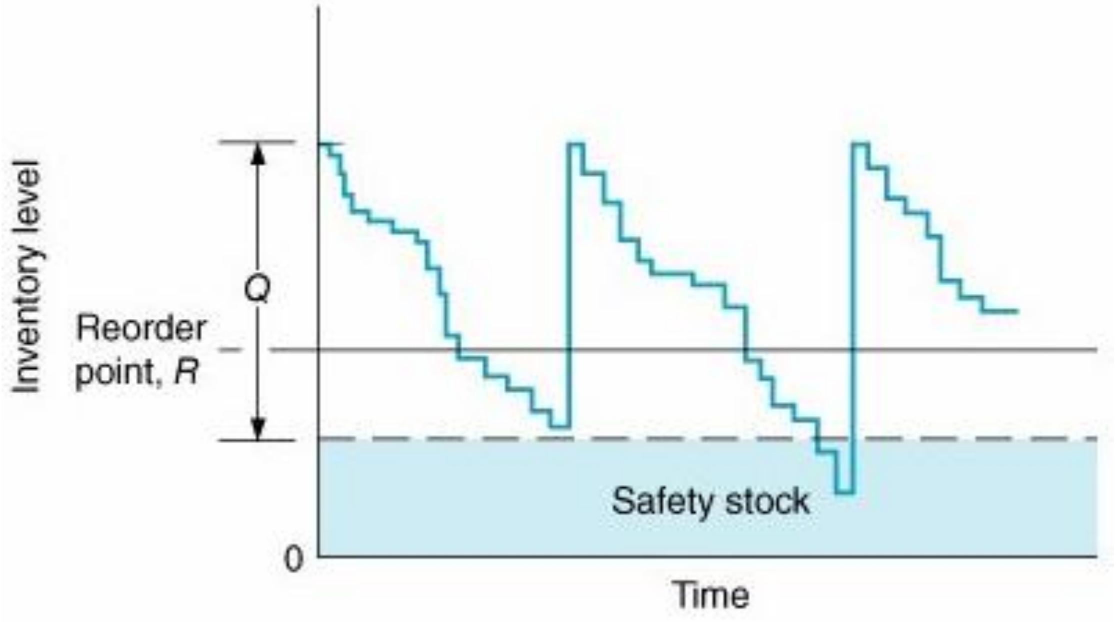
2.1 VARIABLE DEMAND & CONSTANT LEAD TIME

Choosing the appropriate service level policy

- **Cycle-service level**: The desired probability of not running out of stock in any ordering cycle, which begins at the time an order is placed and ends when it arrives.

Reorder point, $R = \bar{d} + SS$

Safety stock, $SS = z \times \sigma_{LT}$



For example, for the Order-Cycle Service Level of 95%, the probability that demand during the lead time will not exceed on-hand inventory at a 95% service level (stockout risk of 5%), i.e. $z = 1.645$

2.1 VARIABLE DEMAND & CONSTANT LEAD TIME

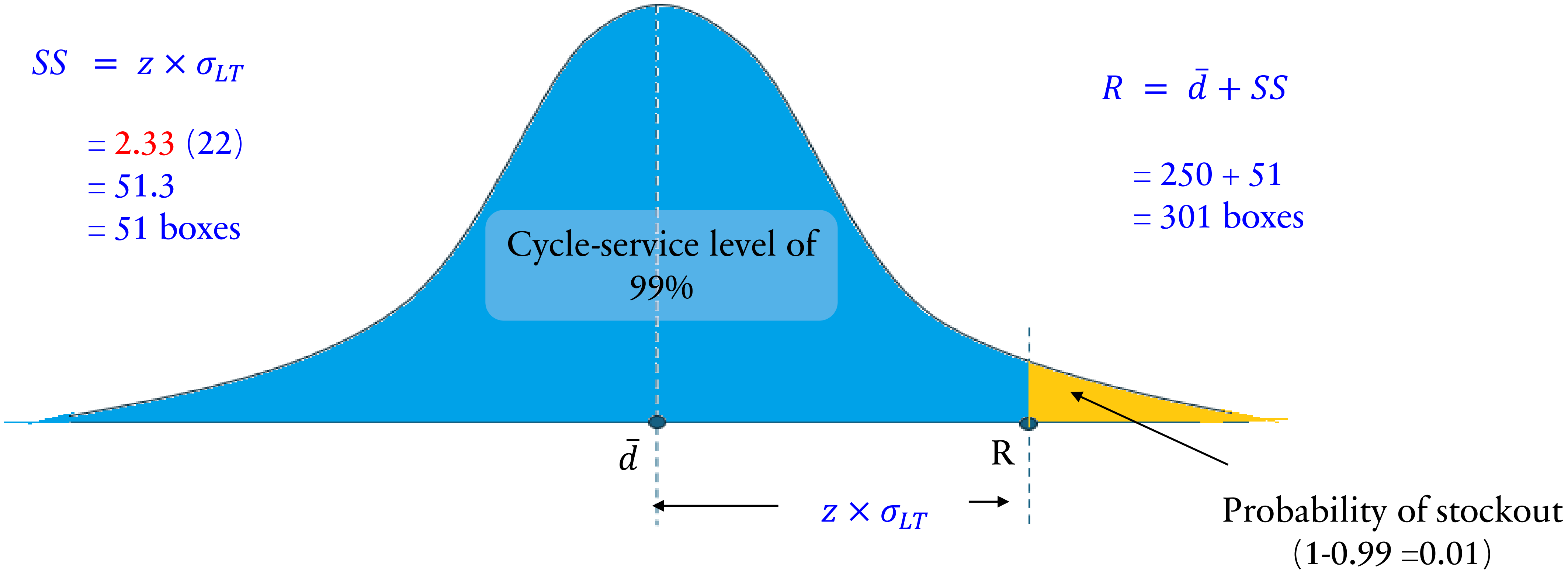
Example 2. The demand for dishwasher detergent during the lead time is normally distributed, with an average of 250 boxes and the standard deviation of the demand during lead time of 22. What safety stock should be carried for a 99 percent cycle-service level? What is R?

$$SS = z \times \sigma_{LT}$$

$= 2.33\ (22)$
$= 51.3$
$= 51$ boxes

$$R = \bar{d} + SS$$

$= 250 + 51$
$= 301$ boxes



2.1 VARIABLE DEMAND & CONSTANT LEAD TIME

Example 2. Suppose that the average demand for bird feeders is 18 units per week with a standard deviation of 5 units. The lead time is constant at 2 weeks. Determine the safety stock and reorder point for a 90 percent cycle-service level.

Given $d = 18$ units per week; $\sigma_d = 5$; $L = 2$ weeks

- Demand distribution for lead time must be developed:

$$\sigma_{LT} = \sigma_d\sqrt{L} = 5\sqrt{2} = 7.1$$

Safety stock, $SS = z \times \sigma_{LT} = 1.28(7.1) = 9.1$ or 9 units

Reorder point $= \bar{d} + SS = d \times L + SS = 18(2) + 9 = 45$ units

2.2 CONSTANT DEMAND & VARIABLE LEAD TIME

When the demand is constant and only the lead time is Variable, then:

$$R = d \cdot \bar{L} + \underbrace{z \cdot d \cdot \sigma_{LT}}_{SS}$$

Example 3. A store sells about 10 digital cameras a day. Lead time For camera delivery is normally distributed with a mean time of 6 days and a standard deviation of 3 days. A 98%Service level is set. Find the reorder point.

Given $d = 10$ units per week; $\bar{L} = 6$; $\sigma_{LT} = 3$

$$R = 10 \cdot 6 + 2.055 \cdot 10 \cdot 3 = 122 \text{ units}$$

2.3 VARIABLE DEMAND & VARIABLE LEAD TIME

Often the case that both are variable; thus, we have slightly more complicated equation

Reorder point = (Average weekly demand × Average lead time) + Safety stock

$$R = (\bar{d} \times \bar{L}) + SS$$

$$SS = z \times \boldsymbol{\sigma_{dLT}}$$

$$\boldsymbol{\sigma_{dLT}} = \sqrt{\bar{L}\sigma_d^2 + (\bar{d})^2 \sigma_{LT}^2}$$

2.3 VARIABLE DEMAND & VARIABLE LEAD TIME

Example 4. A store's most popular item is 9-volt battery. About 150 batteries are sold per day, following a normal distribution with a standard deviation of 16 batteries. Batteries are ordered from an out-of-state distributor; lead time is normally distributed with an average of 5 days a standard deviation of 1 day. To maintain a 95% service level, what ROP is appropriate?

Given $\bar{d}$ = 150 units per day; $\sigma_d = 16$; $\bar{L} = 5$; $\sigma_{LT} = 1$.

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \left(\bar{d}\right)^2 \sigma_{LT}^2} = \sqrt{5 \times 16^2 + 150^2 \times 1^2} \approx 154$$

$$SS = z \times \sigma_{dLT} = 1.65 \times \sigma_{dLT} = 1.65 \times 154 \approx 254 \text{ units}$$

$$R = \left(\bar{d} \times \bar{L}\right) + SS = 150(5) + 254 = 1004 \text{ units}$$

2.3 VARIABLE DEMAND & VARIABLE LEAD TIME

Example 5. Grey Wolf Lodge is a popular 500-room hotel in the North Woods. Managers need to keep close tabs on all room service items, including a special pine-scented bar soap. The daily demand for the soap is 275 bars, with a standard deviation of 30 bars. Ordering cost is $10 and the inventory holding cost is $0.30/bar/year. The lead time from the supplier is 5 days, with a standard deviation of 1 day. The lodge is open 365 days a year.

(a) What is the economic order quantity for the bar of soap?

(b) What should the reorder point be for the bar of soap if management wants to have a 99 percent cycle-service level?

(c) What is the total annual cost for the bar of soap, assuming a Q system will be used?

Given D= 275(365) =100,375 bars of soap; A = $10 per order; h =$0.3 per bar/year

(a) $$EOQ = \sqrt{\frac{2DA}{h}} = \sqrt{\frac{2(100{,}375)(10)}{0.3}} \approx 2{,}587 \text{ bars}$$

2.3 VARIABLE DEMAND & VARIABLE LEAD TIME

b) Given $\bar{d}$ = 275 bars per day; $\sigma_d = 30$ bars; $\bar{L} = 5$ days; $\sigma_{LT} = 1$ day.

$$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + (\bar{d})^2\sigma_{LT}^2} = \sqrt{5 \times 30^2 + 275^2 \times 1^2} \approx 283.06 \text{ bars}$$

Consult the body of the Normal Distribution for 0.9900. The closest value is 0.9901, which corresponds to a z value of 2.33. We calculate the safety stock and reorder point as follows:

$$SS = z \times \sigma_{dLT} = 2.33 \times 283.06 = 659.53 \approx 660 \text{ bars}$$

$$R = (\bar{d} \times \bar{L}) + SS = 275(5) + 660 = 2{,}035 \text{ bars}$$

c) Annual holding cost = $\left(\frac{Q}{2} + SS\right)h = \left(\frac{2{,}587}{2} + 660\right)0.3$

Annual ordering cost = $\frac{D}{Q}A = \frac{100{,}375}{2{,}587} \times 10$

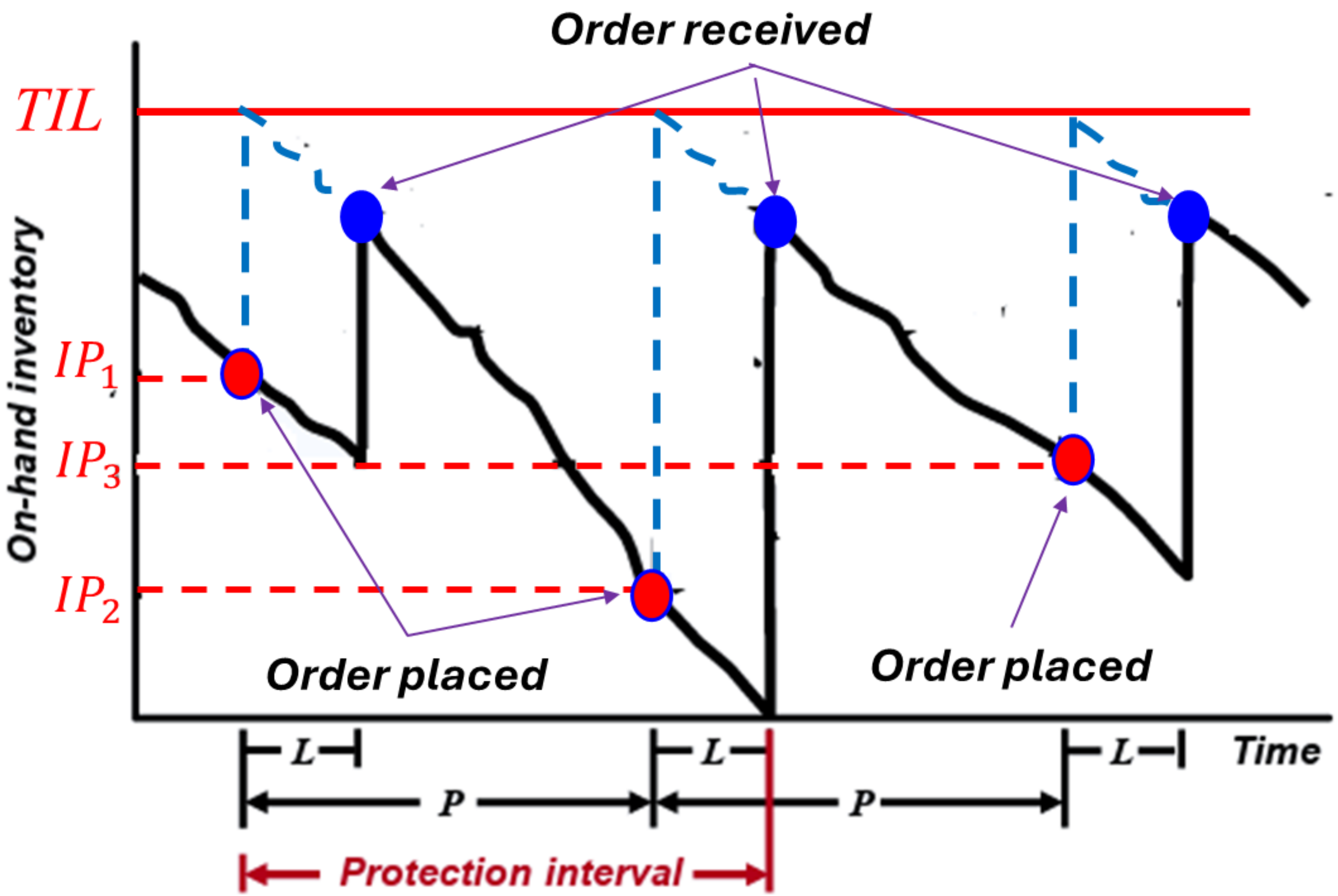
The total annual cost for the Q system = $\left(\frac{Q}{2} + SS\right)h + \frac{D}{Q}A = \$974.05$

3 PERIODIC REVIEW SYSTEM (P SYSTEM)

The Periodic Review System, also called the P System or Fixed Interval Reorder System, is an inventory control method where **orders are placed at fixed, regular time intervals** (e.g., every week or every month), regardless of inventory levels at that moment.

Feature	Description
Review Interval (P)	Orders are placed **every P periods** (e.g., every 4 weeks).
Order Quantity (Q)	Varies each cycle; depends on how much is needed to "top up" to a **target inventory level**.
Target Inventory Level (TI)	A pre-set level that the inventory should reach after each order.
Lead Time (L)	Time taken for the order to arrive after it's placed.
Safety Stock (SS)	Extra stock to cover demand variability during the **protection period** = (P + L).

3 PERIODIC REVIEW SYSTEM (P SYSTEM)



3 PERIODIC REVIEW SYSTEM (P SYSTEM)

Targeted Inventory level

$$TI = d \times (RP + L) + SS$$

d = average period demand; RP = review period (days, wks)

L = lead time (days, wks); $SS = z \times \sigma_{RP+L}$; $\sigma_{RP+L} = \sigma_d \sqrt{RP + L}$

Review Period (RP)

$$RP = \frac{EOQ}{D} \cdot \tau$$

or

$$P = \frac{EOQ}{d}$$

(RP per period)

$EOQ = Q^*$ (optimal lot size)
D = Annual demand
τ = time frame of the demand (e.g. 52 weeks/year if demand is weekly)

3 PERIODIC REVIEW SYSTEM (P SYSTEM)

Example 6. The demand for a bird feeder is normally distributed with a mean of 18 units per week and a standard deviation in weekly demand of 5 units. The lead time is 2 weeks, and the business operates 52 weeks per year. The Q system calls for a system calls for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90 percent. Find review period and target inventory.

- **Find Review Period**

$$D = (18 \text{ units/week})(52 \text{ weeks/year}) = 936 \text{ units}$$

$$RP = \frac{EOQ}{D} \times 52 = \frac{75}{936}(52) = 4.2 \quad \approx 4 \text{ weeks}$$

$$\text{or } \; P = \frac{EOQ}{d} = \frac{75}{18} = 4.2 \quad \approx 4 \text{ weeks}$$

We would review the bird feeder inventory every 4 weeks.

3 PERIODIC REVIEW SYSTEM (P SYSTEM)

- **Find Target Inventory (TI)**

The order-up-to level (TI) when **demand is variable** and lead time is constant will be equal to the average demand during the protection period (**RP** + **LT**) + **Safety Stock**

$$TI = \bar{d} \times (RP + L) + \overbrace{z \times \sigma_{RP+L}}^{SS \text{ for protection interval}}$$

We now find the standard deviation of demand over the protection interval P = (RP + L) = 4 + 2 = 6

$$\therefore \quad \sigma_d \sqrt{RP + L} = 5\sqrt{6} = 12.25 \text{ units}$$

For a 90 percent cycle-service level z = 1.28. The safety stock becomes

$$SS = z \times \sigma_{RP+L} = 1.28\ (12.25) = 15.68 \quad \text{or 16 units}$$

We now have TI = (18 units/week)(6 weeks) + 16 units = 124 units

3 PERIODIC REVIEW SYSTEM (P SYSTEM)

Recall the EOQ formula $Q^* = \sqrt{\frac{2DA}{h}}$

The optimal review period is given by $T = \frac{Q^*}{\bar{d}}$ (the same concept as inventory cycle but with different units)

$$T = \frac{Q^*}{\bar{d}} = \frac{1}{\bar{d}}\sqrt{\frac{2DA}{h}} = \sqrt{\frac{2A}{h \cdot \bar{d}}}$$

Optimal Review Period

$$T = \sqrt{\frac{2A}{h \cdot \bar{d}}}$$

A = Ordering cost per order
h = Holding cost per unit **per unit time**
$\bar{d}$ = Average demand rate in units **per same time**

Maximum Inventory Level

$$S = \bar{d}(T + \bar{L}) + SS$$

$$SS = z \times \sqrt{\sigma_d^2(T + \bar{L}) + \sigma_{LT}^2(\bar{d})^2}$$

3 PERIODIC REVIEW SYSTEM (P SYSTEM)

	Targeted Inventory level	Maximum Inventory Level
Element	$TI = d \times (RP + L) + SS$	$S = \bar{d}(T + \bar{L}) + SS$
Used in	Periodic Review System (P-System)	Also Periodic, but derived from cost-optimised review logic
d or $\bar{d}$	Usually refers to average demand **per period** (empirical or scheduled)	Same — average demand per time unit, but often in **analytical EOQ context**
RP or T	**RP** = fixed review period (often empirical or set by policy)	**T** = optimal review period from EOQ-type formula: $T = \sqrt{\frac{2A}{h \cdot \bar{d}}}$
L or $\bar{L}$	Lead time (often assumed known)	Lead time, possibly based on average or stochastic modeling
Context	Practical, policy-based	Derived from theoretical cost-minimisation
Terminology	"Target Inventory"	"Maximum Inventory Level" or "Order-Up-To Level"

4 PRACTICE QUESTION

VoltHub is a consumer electronics retailer operating in Hamburg. At one of its city-center stores, the expected demand for wireless earbuds is 120 units per month. Each unit costs €50, and the fixed ordering cost is €60 per order. The company applies an annual holding cost rate of 15%. The forecasting MSE for monthly demand is 36. The required service level is 95%.

a) (Reorder Point Policy) Determine the optimal order quantity (EOQ), safety stock, and reorder point assuming a deterministic lead time of 2 weeks.

b) (Periodic Review Policy) Determine the optimal review period T, the maximum inventory level S, and the associated safety stock, assuming the lead time is stochastic with a mean of 0.5 months and a variance of 0.25 months^2.

4 PRACTICE QUESTION

VoltHub is a consumer electronics retailer operating in Hamburg. At one of its city-center stores, the expected demand for wireless earbuds is 120 units per month. Each unit costs $50, and the fixed ordering cost is $60 per order. The company applies an annual holding cost rate of 15%. The forecasting MSE for monthly demand is 36. The required service level is 95%.

a) (Reorder Point Policy) Determine the optimal order quantity (EOQ), safety stock, and reorder point assuming a **deterministic lead time** of 2 weeks.

$\bar{d} = 120$ units/month
$D = 120 \times 12 = 1440$ units/year
$C = \$50$ per unit
$A = \$60$ per order
$I = 15\%$ per year
$h = 15\% \times 50 = \$7.50$ /unit/year
Monthly demand $MSE \approx \sigma_d^2 = 36$
Service level = 95% $\rightarrow z = 1.645$
Time conversions: 1 year = 12 months
$L = 2$ weeks = 0.5 months

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2(60)(1440)}{7.5}} = 151.79 \text{ units}$$

$$SS = z \times \sigma_d \sqrt{L} = 1.645 \times \sqrt{36} \times \sqrt{0.5} = 6.98 \approx 7 \text{ units}$$

$$R = \bar{d} \cdot L + SS = 120 \times 0.5 + 7 = 67 \text{ units}$$

4 PRACTICE QUESTION

b) (Periodic Review Policy) Determine the optimal review period T, the maximum inventory level S, and the associated safety stock, assuming the **lead time is stochastic** with a mean of 0.5 months and a variance of 0.25 months^2.

$\bar{d} = 120$ units/month
$D = 120 \times 12 = 1440$ units/year
$C = \$50$ per unit
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$I = 15\%$ per year
$h = 15\% \times 50 = \$7.50$ /unit/year
Monthly demand $MSE \approx \sigma_d^2 = 36$
Service level = 95% $\rightarrow z = 1.645$
Time conversions: 1 year = 12 months
$\bar{L} = 2$ weeks = 0.5 months
$\sigma_{LT}^2 = 0.25$

$$h = \frac{7.5}{12} = \$0.625 \text{ per unit per month}$$

$$T = \sqrt{\frac{2A}{h \cdot \bar{d}}} = \sqrt{\frac{2(60)}{0.625(120)}} = 1.265 \text{ months}$$

$$SS = z \times \sigma_{dLT} = z \times \sqrt{\sigma_d^2(T + \bar{L}) + \sigma_{LT}^2(\bar{d})^2}$$

$$= 1.645 \times \sqrt{0.5(1.265 + 0.5) + 0.25(120)^2} = 99$$

$$S = \bar{d}(T + \bar{L}) + SS = 120(1.265 + 0.5) + 99 = 203$$