YANSHAN UNIVERSITY

EXAMINATIONS – June/July 2025

Unit Name: Supply Chain Modelling and Optimisation

Duration: 2 hours

Total Marks: 100

Calculator: Yes, any hand-held calculator approved by Yanshan University

THIS IS A CLOSED BOOK EXAM

IMPORTANT INFORMATION

Mobile phones or any other devices capable of communicating information are prohibited during examinations.

Electronic Organizers/PDAs (with the exception of calculators) or any other similar devices capable of storing restricted text or restricted information are prohibited during examinations.

Other Information: This paper contains **4** questions

Attempt as many questions as you can.

All working must be shown

Surname: ______________________________

Given Name: ______________________________

Student Number: ______________________________

Question 1 (25 marks)

A retailer must determine optimal warehouse locations to distribute a single commodity (e.g., bottled water) to three customer zones. Two warehouse locations are available (W1 and W2). Given warehouse data, customer demand and transportation cost from warehouse to customer as shown in the table below.

Table 1: Warehouse Transportation Costs, Capacities, and Fixed Costs

Warehouse	**Transportation cost ($/unit)**			**Capacity (units)**	**Fixed cost ($)**
	Zone A	Zone B	Zone C		
W1	4	3	5	300	1000
W2	2	4	3	250	800
Demand	150	120	80		

Let

- Facility opening variables:

$$z_j = \begin{cases} 1, & \text{if warehouse } j \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$$

- Commodity flow variables: y_{jk} which is the quantity shipped from warehouse j to customer zone k
- Warehouse capacities: K_j
- Fixed warehouse opening costs: f_j
- Demand at customer zones: D_k
- Transportation costs per unit: c_{jk}

The Optimization model is

$$\min Z = \sum_{j \in V_1} f_j z_j + \sum_{j \in V_1} \sum_{k \in V_2} c_{jk} y_{jk}$$

subject to

- Demand Satisfaction Constraints

$$y_{W1,k} + y_{W2,k} = D_k, \quad \forall k \in \{A, B, C\}$$

- Warehouse Capacity Constraints

$$y_{W1,A} + y_{W1,B} + y_{W1,C} \leq 300 z_{W1}$$

$$y_{W2,A} + y_{W2,B} + y_{W2,C} \leq 250 z_{W2}$$

- Facility Binary Constraints $z_j \in \{0, 1\}, \quad \forall j \in \{W1, W2\}$

- Non-negativity Constraints $y_{jk} \geq 0, \quad \forall j, k$

Tasks:

a) What are the decision variables in the given warehouse location problem? (5 marks)

b) Explain the objective function used in the optimization model. What components contribute to the total cost? (6 marks)

c) Why are binary variables z_j used in the model? (2 marks)

d) What is the role of the non-negativity constraint in this model? (2 marks)

e) What changes would be necessary to model a scenario where each customer must be served by exactly one warehouse? (10 marks)

Question 2 (25 marks)

Sarath is a Malaysia-based distributor of Korean appliances. The sales volume of portable TV sets during the last 12 weeks in Kuala Lumpur is shown in Table 2. Sarath applies several forecasting methods to forecast future demand.

Table 2: Number of portable TV sets sold in the last 12 weeks.

Time period	Quantity	Time period	Quantity
1	1180	7	1162
2	1176	8	1163
3	1185	9	1180
4	1163	10	1170
5	1172	11	1161
6	1172	12	1177

Table 3: Week, Quantity, X^2, and $X \times Y$ Calculation

	X	Y	X^2	$X \times Y$
	1	1180	1	1180
	2	1176	4	2352
	3	1185	9	3555
	4	1163	16	4652
	5	1172	25	5860
	6	1172	36	7032
	7	1162	49	8134
	8	1163	64	9304
	9	1180	81	10620
	10	1170	100	11700
	11	1161	121	12771
	12	1177	144	14124
Total	78	14061	650	91284

a) Using information in Table 3, fit a linear regression model to the historical sales data in the form of $\hat{y}_t = a + bx$ and forecast future demand. (10 marks)

Hint: $b = \dfrac{\sum XY - n\bar{x}\bar{y}}{\sum X^2 - n\bar{x}^2}$ and $a = \bar{y} - b\bar{x}$ where $\bar{x} = \dfrac{\sum X}{n}$ and $\bar{y} = \dfrac{\sum Y}{n}$.

b) Using a 3-week moving average, calculate future demand by showing all the detailed calculations. (3 marks)

c) Using a smoothing constant $\alpha = 0.3$, the process begins by setting $\hat{y}_2 = y_1 = 1180$. The exponential smoothing (ES) forecasts for subsequent weeks are summarised in Table 3.

Table 4: Exponential Smoothing Forecasts

Week	Quantity	ES Forecast
1	1180	-
2	1176	1180.00
3	1185	1178.80
4	1163	1180.66
5	1172	1175.36
6	1172	1174.35
7	1162	1173.65
8	1163	1170.15
9	1180	1168.01
10	1170	1171.61
11	1161	1171.12
12	1177	1168.09

– Show that the most recent observations play a more significant role than those from the distant past. (3 marks)

- Forecast future demand by showing all the detailed steps. (4 marks)

d) Which method would you recommend for forecasting of portable TV set sales? Justify your recommendation by considering factors such as responsiveness to recent changes, smoothing of random fluctuations, and the underlying data pattern. (5 marks)

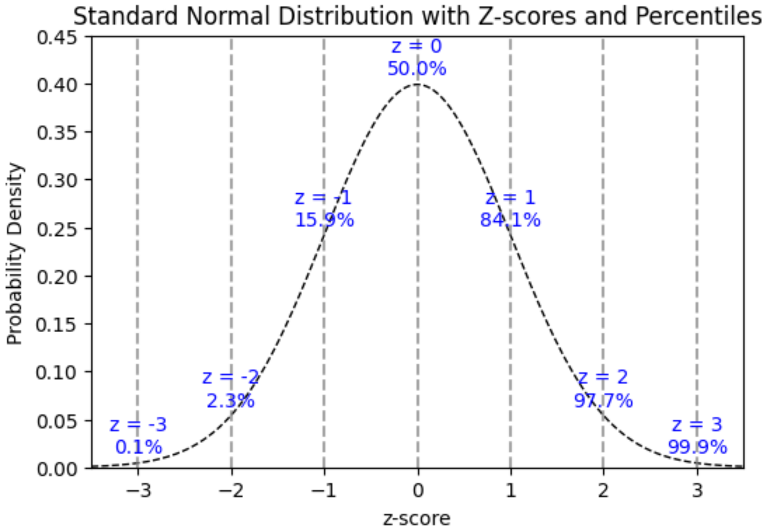
Question 3 (25 marks)

Golden Food distributes tinned foodstuffs in Great Britain. In a warehouse located in Birmingham, the demand rate d for tomato puree is 400 pallets a month. The value of a pallet is $C = €2500$ and the annual interest rate I is 14.5% (including warehousing costs). Issuing an order costs €30. The replenishment rate is 40 pallets per day. Assuming that the number of workdays in a month equals 20. Determine optimal lot size Q^*, cycle length T and total cost TC if shortages are not allowed.

Hint: $Q^* = \sqrt{\dfrac{2DA}{h(1-d/r)}}, \quad TC(Q^*) = CD + \dfrac{D}{Q^*}A + \dfrac{Q^*}{2}h\left(1 - \dfrac{d}{r}\right)$

Question 4 (25 marks)

Papier is a French retail chain. At the outlet located in downtown Lyon, the expected demand for mouse pads is 45 units per month. The value of an item in stock is €4, and the fixed reorder cost is equal to €30. The annual interest rate is 20%. The demand forecasting MSE is 25. The service level is required to be equal to 97.7%.



a) For the **reorder point policy**, determine the optimal lot size (EOQ), safety stock, and reorder point assuming a **deterministic lead time of 1 month**. (12 marks)
Hint:

- Economic Order Quantity (EOQ): $Q = \sqrt{\frac{2AD}{h}}$
- Safety stock: $\sigma_d = \sqrt{MSE}$; $SS = z_\alpha \cdot \sigma_d \cdot \sqrt{L}$
- Reorder Point (ROP): $ROP = \bar{d} \cdot L + SS$

b) For the **periodic review policy**, determine the optimal review period T, the maximum inventory level S, and the associated safety stock. Assume the **lead time is stochastic** with a mean of 1 month and a variance of 1.5 months².
(13 marks)

Hint

Review period $T = \sqrt{\frac{2A}{h_{\text{monthly}} \cdot \bar{d}}}$

Safety stock $SS = z_\alpha \cdot \sqrt{\sigma_d^2(T + \bar{L}) + \sigma_L^2 \cdot \bar{d}^2}$

Maximum inventory level $S = \bar{d}(T + \bar{L}) + SS$

END OF EXAMINATION

Notations

- d: demand rate
- D: annual demand
- K: ordering cost
- H, h: holding cost
- C: unit cost
- L: lead time
- P: review period
- SS: safety stock
- z: service factor
- σ_d: demand std dev
- σ_L: lead time std dev
- r: production rate
- p: shortage cost

1. EOQ & EPQ with Shortages

a) EOQ with Planned Shortages

$$Q^* = \sqrt{\frac{2DK}{H} \cdot \frac{H+p}{p}}$$

$$M^* = \frac{p}{H+p} Q^* \qquad S^* = \frac{H}{H+p} Q^*$$

$$TC(Q, M, S) = \frac{DK}{Q} + \frac{HM^2}{2Q} + \frac{pS^2}{2Q}$$

b) EPQ with Planned Shortages

$$Q^* = \sqrt{\frac{2DK}{H} \cdot \frac{r}{r-d} \cdot \frac{H+p}{p}}$$

$$M^* = \frac{p}{H+p} \left(1 - \frac{d}{r}\right) Q^*$$

$$S^* = \frac{H}{H+p} \left(1 - \frac{d}{r}\right) Q^*$$

$$TC(Q, M, S) = \frac{DK}{Q} + \frac{HM^2}{2Q} + \frac{pS^2}{2Q}$$

2. Model with Discount

a) All-Units Discount

EOQ at price level i:

$$Q_i^* = \sqrt{\frac{2DK}{H_i}}, \quad H_i = I \times C_i$$

Total cost:

$$TC_i(Q) = DC_i + \frac{DK}{Q} + \frac{H_i Q}{2}$$

Feasibility condition:

$$q_1 \le Q_i^* \le q_{i+1}$$

Optimal solution:

$$Q^* = \arg\min_i \{TC_i(Q)\}$$

b) Incremental Quantity Discount

$$Q_i^* = \sqrt{\frac{2\left(R_{i-1} - C_i q_{i-1} + K\right) D}{IC_i}}$$

$$R_i = C_1(q_2 - q_1) + C_2(q_3 - q_2) + \ldots + C_{i-1}(q_i - q_{i-1})$$

Total cost:

$$TC(Q) = DC_i + (R_i - C_i q_i + K)\frac{D}{Q} + \frac{Q}{2}(IC_i) + I\frac{R_i - C_i q_i}{2}$$

3. Multicommodity Models

a) Independent Ordering

$$Q_i^* = \sqrt{\frac{2D_i K_i}{IC_i}}$$

b) Joint Replenishment (All Items)

Combined ordering cost:

$$K^* = K + \sum_i K_i$$

Optimal ordering frequency:

$$n^* = \sqrt{\frac{\sum_i D_i IC_i}{2K^*}}$$

Lot size:

$$Q_i = \frac{D_i}{n^*}$$

Total cost:

$$TC(n) = nK^* + \sum_i \frac{D_i IC_i}{2n}$$

c) Joint Ordering for some Products
Base frequency:

$$n = \sqrt{\frac{\sum_i IC_i m_i D_i}{2\left(K + \sum_i \frac{K_i}{m_i}\right)}},\ m_i = \lceil \overline{n}/\overline{\overline{n}}_i \rceil$$

where $\overline{n}_i = \sqrt{\frac{IC_i D_i}{2(K + K_i}}$; $\overline{n} = \max\{\overline{n}_i\}$

and

$$\overline{\overline{n}}_i = \sqrt{\frac{IC - iD_i}{2K_i}}.$$

Item frequency:

$$n_i = \frac{n}{m_i}$$

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4. Stochastic Inventory (Newsvendor)
Cost Parameters:

$$C_o = C - v, \quad C_u = r - C$$

Critical Ratio:

$$CR = \frac{C_u}{C_u + C_o} = \frac{r - C}{r - v}$$

Optimal Order Rule:

$$P(D \leq Q^*) = CR$$

Discrete Demand:

$$P(D \leq Q^*) \geq CR$$

Continuous Demand:

$$F(Q^*) = CR$$

Normal Distribution:

$$Q^* = \mu + z\sigma$$

Uniform Distribution:

$$Q^* = a + (b - a)CR$$

Stockout Probability:

$$P(\text{stockout}) = 1 - F(Q^*)$$

Poisson Distribution:

$$P(X = n) = \frac{e^{-\lambda}\lambda^n}{n!}$$

$$F(n) = \sum_{k=0}^{n} \frac{e^{-\lambda}\lambda^k}{k!}$$

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5. Continuous Review (Q, R) System
a) ROP With No Uncertainty

$$R = d \times L$$

b) ROP With Demand Uncertainty

$$ROP = d \times L + SS$$

$$SS = z\sigma_L$$

$$\sigma_L = \sigma_d \sqrt{L}$$

c) Variability Cases
Constant demand, variable lead time

$$R = d \times \bar{L} + SS$$

$$SS = z \times d \times \sigma_{LT}$$

Variable demand, variable lead time

$$ROP = \bar{d} \times \bar{L} + SS$$

$$SS = z\sqrt{\sigma_d^2 \bar{L} + (\bar{d})^2 \sigma_L^2}$$

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6. Periodic Review (P) System
a) TIL With No Uncertainty

$$TIL = d \times (P + L)$$

b) TIL With Uncertainty

$$TIL = d \times (P + L) + SS$$

$$SS = z\sigma_{P+L}$$

$$\sigma_{P+L} = \sigma\sqrt{P + L}$$

Service Level

$z \approx 1.645$ for 95%

Optimal Lot Size

$$Q^* = \sqrt{\frac{2DK}{H}}$$

Alternative form:

$$Q^* = \sqrt{\frac{2dK}{h}}$$

Periodic Review:

$$P^* = \sqrt{\frac{2K}{hd}}$$

$$TIL = d(P^* + L) + SS$$

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7. Total Cost

Continuous Review:

$$TC(Q) = \frac{DK}{Q} + H\left(\frac{Q}{2} + SS\right)$$

Periodic Review:

$$TC(Q) = \frac{DK}{dP} + H\left(\frac{dP}{2} + SS\right)$$

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8. (Q, R) Model with Backorders

Lead-Time Demand

$$\mu = D\tau, \quad \sigma = \sigma_y\sqrt{\tau}$$

Service Level Factor

$$z = \frac{R - \mu}{\sigma}, \quad F(z) = P(D(\tau) \le R)$$

Expected Shortage per Cycle

$$n(R) = \sigma L(z)$$

Total Cost

$$TC(Q, R) = \frac{DK}{Q} + H\left(R - \mu + \frac{Q}{2}\right) + \frac{pD}{Q}n(R)$$

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9. Optimality Conditions

Reorder point:

$$F(R) = 1 - \frac{HQ}{pD}$$

Order quantity:

$$Q^* = \sqrt{\frac{2D}{H}\left(K + p\,n(R)\right)}$$

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10. Iterative Algorithm

$$Q_0 = \sqrt{\frac{2DK}{H}}$$

$$F(R_i) = 1 - \frac{HQ_i}{pD}$$

$$Q_{i+1} = \sqrt{\frac{2D}{H}\left(K + p\,n(R_i)\right)}$$

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11. Service Levels

- Type I (cycle service)

$$\alpha = F(z)$$

- Type II (fill rate)

$$\beta = 1 - \frac{n(R)}{Q}$$

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12. Stockout Relation

$$n(R) = (1 - \beta)Q$$

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13. Alternative Form (Target β)

$$Q = \frac{n(R)}{1 - F(R)} + \sqrt{\frac{2DK}{H} + \left(\frac{n(R)}{1 - F(R)}\right)^2}$$

14. Z-Chart & Loss Function

Standard Normal

$$Z \sim N(0,1), \quad F(z) = P(Z \le z)$$

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

Loss Function

$$L(z) = \phi(z) - z\big(1 - F(z)\big)$$

$$L(z) = \int_z^{\infty} (x - z)\phi(x)\, dx$$

Key Relationships

$$P(Z > z) = 1 - F(z)$$

$$F(-z) = 1 - F(z)$$

Inventory Use

$$R = \mu + z\sigma$$

$$n(R) = \sigma L(z)$$

Interpretation

- $F(z)$: service level
- $L(z)$: expected shortage factor
- Lost sales $= \sigma L(z)$

Z	$F(Z)$	$L(Z)$	Z	$F(Z)$	$L(Z)$	Z	$F(Z)$	$L(Z)$	Z	$F(Z)$	$L(Z)$
-3.00	0.0013	3.000	-1.48	0.0694	1.511	0.04	0.5160	0.379	1.56	0.9406	0.026
-2.96	0.0015	2.960	-1.44	0.0749	1.474	0.08	0.5319	0.360	1.60	0.9452	0.023
-2.92	0.0018	2.921	-1.40	0.0808	1.437	0.12	0.5478	0.342	1.64	0.9495	0.021
-2.88	0.0020	2.881	-1.36	0.0869	1.400	0.16	0.5636	0.324	1.68	0.9535	0.019
-2.84	0.0023	2.841	-1.32	0.0934	1.364	0.20	0.5793	0.307	1.72	0.9573	0.017
-2.80	0.0026	2.801	-1.28	0.1003	1.327	0.24	0.5948	0.290	1.76	0.9608	0.016
-2.76	0.0029	2.761	-1.24	0.1075	1.292	0.28	0.6103	0.274	1.80	0.9641	0.014
-2.72	0.0033	2.721	-1.20	0.1151	1.256	0.32	0.6255	0.259	1.84	0.9671	0.013
-2.68	0.0037	2.681	-1.16	0.1230	1.221	0.36	0.6406	0.245	1.88	0.9699	0.012
-2.64	0.0041	2.641	-1.12	0.1314	1.186	0.40	0.6554	0.230	1.92	0.9726	0.010
-2.60	0.0047	2.601	-1.08	0.1401	1.151	0.44	0.6700	0.217	1.96	0.9750	0.009
-2.56	0.0052	2.562	-1.04	0.1492	1.117	0.48	0.6844	0.204	2.00	0.9772	0.008
-2.52	0.0059	2.522	-1.00	0.1587	1.083	0.52	0.6985	0.192	2.04	0.9793	0.008
-2.48	0.0066	2.482	-0.96	0.1685	1.050	0.56	0.7123	0.180	2.08	0.9812	0.007
-2.44	0.0073	2.442	-0.92	0.1788	1.017	0.60	0.7257	0.169	2.12	0.9830	0.006
-2.40	0.0082	2.403	-0.88	0.1894	0.984	0.64	0.7389	0.158	2.16	0.9846	0.005
-2.36	0.0091	2.363	-0.84	0.2005	0.952	0.68	0.7517	0.148	2.20	0.9861	0.005
-2.32	0.0102	2.323	-0.80	0.2119	0.920	0.72	0.7642	0.138	2.24	0.9875	0.004
-2.28	0.0113	2.284	-0.76	0.2236	0.889	0.76	0.7764	0.129	2.28	0.9887	0.004
-2.24	0.0125	2.244	-0.72	0.2358	0.858	0.80	0.7881	0.120	2.32	0.9898	0.003
-2.20	0.0139	2.205	-0.68	0.2483	0.828	0.84	0.7995	0.112	2.36	0.9909	0.003
-2.16	0.0154	2.165	-0.64	0.2611	0.798	0.88	0.8106	0.104	2.40	0.9918	0.003
-2.12	0.0170	2.126	-0.60	0.2743	0.769	0.92	0.8212	0.097	2.44	0.9927	0.002
-2.08	0.0188	2.087	-0.56	0.2877	0.740	0.96	0.8315	0.090	2.48	0.9934	0.002
-2.04	0.0207	2.048	-0.52	0.3015	0.712	1.00	0.8413	0.083	2.52	0.9941	0.002
-2.00	0.0228	2.008	-0.48	0.3156	0.684	1.04	0.8508	0.077	2.56	0.9948	0.002
-1.96	0.0250	1.969	-0.44	0.3300	0.657	1.08	0.8599	0.071	2.60	0.9953	0.001
-1.92	0.0274	1.930	-0.40	0.3446	0.630	1.12	0.8686	0.066	2.64	0.9959	0.001
-1.88	0.0301	1.892	-0.36	0.3594	0.605	1.16	0.8770	0.061	2.68	0.9963	0.001
-1.84	0.0329	1.853	-0.32	0.3745	0.579	1.20	0.8849	0.056	2.72	0.9967	0.001
-1.80	0.0359	1.814	-0.28	0.3897	0.554	1.24	0.8925	0.052	2.76	0.9971	0.001
-1.76	0.0392	1.776	-0.24	0.4052	0.530	1.28	0.8997	0.047	2.80	0.9974	0.001
-1.72	0.0427	1.737	-0.20	0.4207	0.507	1.32	0.9066	0.044	2.84	0.9977	0.001
-1.68	0.0465	1.699	-0.16	0.4364	0.484	1.36	0.9131	0.040	2.88	0.9980	0.001
-1.64	0.0505	1.661	-0.12	0.4522	0.462	1.40	0.9192	0.037	2.92	0.9982	0.001
-1.60	0.0548	1.623	-0.08	0.4681	0.440	1.44	0.9251	0.034	2.96	0.9985	0.000
-1.56	0.0594	1.586	-0.04	0.4840	0.419	1.48	0.9306	0.031	3.00	0.9987	0.000
-1.52	0.0643	1.548	0.00	0.5000	0.399	1.52	0.9357	0.028			