YSU SUPPLY CHAIN MODELLINGAND OPTIMISATION (2025)

SOLUTIONS

General Marking Guidlines

1. The writings in black are from exam questions; the writings in blue are the solutions/answers to the questions; the writings in red are marks for the questions/sub-questions

   (黑色文字来自考试题目；蓝色文字为答案；红色文字为分数).

2. For each question/sub-question with multiple work steps, if the final answer is incorrect due to an error in a certain step, only deduct the score for that step. That is, do not deduct scores for carried-over errors unless new errors occur or the remaining work becomes simpler.

   (对于包含多个工作步骤的问题/子问题，如果由于某一步骤中的错误导致最终答案不正确，则仅扣除该步骤的分数。也就是说，除非出现新的错误或剩余工作变得更简单，否则不应因延续的错误而扣分。)

3. If the final answer to a question/sub-question is correct but the working/presentation is different from that given in these standard solutions, up to the full mark may be awarded for that question/sub-question unless there are any errors or the work does not follow the specific method/procedure required in the question

   (如果问题/子问题的最终答案正确，但其运算/表达方式与这些标准答案中给出的不同，则除非运算存在错误或没有遵循问题所要求的特定方法，该问题/子问题也可获得满分。)

Question 1 (20 marks)

a) Decision Variables: **(5 marks)**

- $z_j \in \{0,1\}$ be binary variable, equals 1 if warehouse $j \in \{W1, W2\}$ is opened, 0 otherwise. **(2.5)**
- $y_{jk} \geq 0$ be quantity of product shipped from warehouse j to customer zone $k \in \{A, B, C\}$. **(2.5)**

b) Objective Function Explanation: **(6 marks)**

The objective function:

$$\min Z = \sum_{j \in V_1} f_j z_j + \sum_{j \in V_1} \sum_{k \in V_2} c_{jk} y_{jk} \quad \textbf{(2+2)}$$

minimizes the total cost, which consists of:

- Fixed warehouse opening costs: $f_j z_j$ **(1)**
- Variable transportation costs from warehouses to customer zones: $c_{jk} y_{jk}$ **(1)**

c) Binary Variable Role: **(2 marks)**

The binary variables z_j indicate whether a warehouse is open or not. If $z_j = 0$, the corresponding warehouse cannot ship any units (enforced by the capacity constraint), hence modelling the fixed cost only when a warehouse is operational.

d) Non-negativity Constraints: **(2 marks)**

The constraint $y_{jk} \geq 0$ ensures that the quantity shipped from warehouse j to zone k is not negative, which is consistent with real-world logistics and physical quantities.

e) Modification for Exclusive Supply: **(10 marks)**

To ensure each customer is served by exactly one warehouse, introduce new binary variables $x_{jk} \in \{0,1\}$ where:

$$\sum_{j \in \{W1, W2\}} x_{jk} = 1, \quad \forall k \in \{A, B, C\}$$

Then relate y_{jk} to x_{jk} using:

$$y_{jk} = D_k \cdot x_{jk}$$

The update optimisation model is:

$$\min Z = \sum_{j \in V_1} f_j z_j + \sum_{j \in V_1} \sum_{k \in V_2} c_{jk} D_k x_{jk} \quad \textbf{(2 + 3)}$$

subject to

$$\sum_{j \in \{W1, W2\}} x_{jk} = 1, \quad \forall k \in \{A, B, C\} \quad \textbf{(2)}$$

$$\sum_{k \in \{A,B,C\}} D_k x_{jk} \leq K_j z_j \quad \forall j \in \{W_1, W_2\} \quad \textbf{(2)}$$

$$z_j, x_{jk} \in \{0,1\} \quad \textbf{(1)}$$

Question 2 (25 marks)

a) **Linear Regression Forecasting** **(10 marks)**

We are given the formula:

$$\hat{y}_t = a + bx$$

Where:

$$b = \frac{\sum XY - n\bar{x}\bar{y}}{\sum X^2 - n\bar{x}^2}, \quad a = \bar{y} - b\bar{x}$$

From the table:

$$\sum X = 78, \quad \sum Y = 14061, \quad \sum X^2 = 650, \quad \sum XY = 91284, \quad n = 12$$

Means:

$$\bar{x} = \frac{78}{12} = 6.5, \quad \bar{y} = \frac{14061}{12} \approx 1171.75 \quad \textbf{(2)}$$

Compute b:

$$b = \frac{91284 - 12 \cdot 6.5 \cdot 1171.75}{650 - 12 \cdot (6.5)^2} = \frac{91284 - 91486.5}{650 - 507} = \frac{-112.5}{143} \approx -0.78671 \quad \textbf{(2)}$$

Compute a:

$$a = 1171.75 - (-0.78671) \cdot 6.5) = 1171.75 + 5.1136 \approx 1176.86 \quad \textbf{(2)}$$

Regression Equation:

$$\hat{y}_t = 1176.86 - 0.7867x \quad \textbf{(2)}$$

Forecast for Week 13:

$$\hat{y}_{13} = 1176.86 - 0.7867 \cdot 13 \approx 1176.86 - 10.2271 = \boxed{1166.63} \quad \textbf{(2)}$$

b) **3-Week Moving Average Forecast** **(3 marks)**

$$\text{SMA}_{13} = \frac{y_{10} + y_{11} + y_{12}}{3} = \frac{1170 + 1161 + 1177}{3} = \frac{3508}{3} = \boxed{1169.33} \quad \textbf{(3)}$$

c) **Exponential Smoothing** **(7 marks)**

i. **Explanation:** **(3 marks)**

Exponential smoothing applies a weight $\alpha = 0.3$ to the most recent observation and $(1 - \alpha)$ to the previous forecast:

$$\hat{y}_{t+1} = \alpha y_t + (1 - \alpha)\hat{y}_t \quad \textbf{(1.5)}$$

This means recent observations influence the forecast more heavily than older data. For example:

$$\hat{y}_{12} = 1168.09, \quad \hat{y}_2 = 1180.00 \quad \textbf{(1.5)}$$

Despite earlier forecasts being higher, the model gradually adjusts downward in response to recent lower actual sales (e.g., 1161 at week 11).

ii. **Forecast for Week 13:** (4 marks)

Given:

$$y_{12} = 1177, \quad \hat{y}_{12} = 1168.09, \quad \alpha = 0.3 \quad \textbf{(2)}$$

$$\hat{y}_{13} = 0.3 \cdot 1177 + 0.7 \cdot 1168.09 = 353.1 + 817.663 = \boxed{1170.76} \quad \textbf{(2)}$$

d) **Recommendation** **(5 marks)**

Summary of Forecasts for Week 13:

- Linear regression: 1162.54
- 3-week moving average: 1169.33
- Exponential smoothing: 1170.76

Recommendation:

The sales data show minor fluctuations with no strong upward or downward trend. Therefore, we conclude that

- **Linear regression** underestimates future demand by enforcing a linear downward trend, which doesn't reflect the slight cyclical variation. **(1)**
- **Moving average** smooths data well but may lag behind rapid changes. **(1)**
- **Exponential smoothing** is most responsive to recent changes and provides a balanced, accurate forecast. **(1)**

Thus, exponential smoothing is recommended for forecasting sales due to its adaptability and sensitivity to recent patterns. **(1)**

Recommended method: Exponential Smoothing, Forecast = 1170.76 units **(1)**

Question 3 (25 marks)

Given:

- Monthly demand: $d = 400$ pallets
- Value of a pallet: $C = €2500$
- Annual interest rate: $I = 14.5\%$
- Ordering cost: $A = €30$
- Workdays in a month: 20
- Replenishment rate: $r = 40$ pallets/day

Derived Parameters

- Daily demand:

$$\frac{400}{20} = 20 \text{ pallets/day} \quad \textbf{(2)}$$

- Annual demand:

$$D = 400 \times 12 = 4800 \text{ pallets/year} \quad \textbf{(2)}$$

- Holding cost per unit per year:

$$h = 2500 \times 0.145 = €362.50 \quad \textbf{(2)}$$

Optimal Lot Size Q^* (with finite production rate)

$$Q^* = \sqrt{\frac{2DA}{h(1-d/r)}} = \sqrt{\frac{2 \cdot 4800 \cdot 30}{362.5 \cdot (1 - 20/40)}} = \sqrt{\frac{288000}{181.25}} = \sqrt{1587.59} = \boxed{39.86 \text{ pallets}} \quad \textbf{(4)}$$

Cycle Length T

$$T = \frac{Q^*}{d} = \frac{39.86}{400} = 0.10 \text{ months} = \boxed{2 \text{ workdays}} \quad \textbf{(4)}$$

Total Annual Cost TC

$$TC(Q) = CD + \frac{D}{Q^*}A + \frac{Q^*}{2}h\left(1 - \frac{d}{r}\right)$$

$$CD = 2500 \cdot 4800 = €12{,}000{,}000 \quad \textbf{(3)}$$

$$\text{Ordering cost} = \frac{4800}{39.86} \cdot 30 = €361.28 \quad \textbf{(3)}$$

$$\text{Holding cost} = \frac{39.86}{2} \cdot 362.5 \cdot \left(1 - \frac{20}{40}\right) = 19.93 \cdot 362.5 \cdot 0.5 = €11{,}863.68 \quad \textbf{(3)}$$

$$TC = 12{,}000{,}000 + 361.28 + 11{,}863.68 = \boxed{€12{,}007{,}224.96} \quad \textbf{(2)}$$

Question 4 (25 marks)

Given:

- Monthly demand: $\bar{d} = 45$ units
- Unit cost: €4
- Fixed ordering cost: €30
- Annual interest rate: 20%
- Forecast MSE (demand): 25
- Mean lead time: $\bar{L} = 1$ month
- Lead time MSE: $\sigma_L^2 = 1.5$ months^2
- Service level: 97.7% $\Rightarrow z_\alpha = 2.00$

a) Reorder Point Policy (Deterministic Lead Time) (12 marks)

- **Annual holding cost:**

$$h = 4 \times 0.20 = 0.80 \text{ €/unit/year} \quad \textbf{(1)}$$

- **Annual demand:**

$$D = 45 \times 12 = 540 \text{ units/year} \quad \textbf{(1)}$$

- **Economic Order Quantity (EOQ):**

$$Q = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2 \cdot 30 \cdot 540}{0.80}} = \sqrt{40500} = \boxed{201.25 \text{ units}} \quad \textbf{(3)}$$

- **Safety stock:**

$$\sigma_d = \sqrt{25} = 5 \quad \textbf{(2)}$$

$$SS = z_\alpha \cdot \sigma_d \cdot \sqrt{\bar{L}} = 2.00 \cdot 5 \cdot \sqrt{1} = \boxed{10 \text{ units}} \quad \textbf{(2)}$$

- **Reorder Point (ROP):**

$$ROP = \bar{d} \cdot L + SS = 45 + 10 = \boxed{55 \text{ units}} \quad \textbf{(3)}$$

b) Periodic Review Policy (Stochastic Lead Time) (13 marks)

- **Monthly holding cost:** $h_{\text{monthly}} = \dfrac{0.80}{12} = 0.0667$ €/unit/month **(2)**

- **Review period T:**

$$T = \sqrt{\frac{2A}{h_{\text{monthly}} \cdot \bar{d}}} = \sqrt{\frac{2 \cdot 30}{0.0667 \cdot 45}} = \sqrt{20} = \boxed{4.47 \text{ months}} \quad \textbf{(3)}$$

- **Safety stock:**

$$\begin{aligned} SS = z_\alpha \cdot \sqrt{\sigma_d^2(T + \bar{L}) + \sigma_L^2 \cdot \bar{d}^2} &= 2 \times \sqrt{25(4.47 + 1) + 1.5 \times 45^2} \\ &= 2 \times \sqrt{683.75 + 3037.50} \\ &= 2 \times \sqrt{3721.25} = \boxed{112.42 \text{ units}} \quad \textbf{(4)} \end{aligned}$$

- **Maximum inventory level S:**

$$S = \bar{d}(T + \bar{L}) + SS = 45 \cdot 5.47 + 112.42 = \boxed{358.67 \text{ units}} \quad \textbf{(2)}$$

END OF EXAMINATION SOLUTIONS