

05115130 SCMO (June 2026 Class)

Name (in English): \_\_\_\_\_

Assignment (20%)

Due: Sat 13 June (5:10 PM)

Student ID: \_\_\_\_\_

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### Question 1 (20 marks)

a) Consider the optimisation problem

$$\begin{aligned} \text{Minimise } & f(x_1, x_2) = x_1^2 + x_2^2 + x_1x_2 - 5x_2 \\ \text{subject to } & x_1 + 2x_2 = 5, \\ & 2x_1 - x_2 = 10, \\ & x_1 + x_2 \leq 4. \end{aligned}$$

- i) Write down the Lagrangian formula  $L(x_1, x_2, \lambda_1, \lambda_2, \mu)$ . (1 mark)
- ii) Determine all constraints of  $L(x_1, x_2, \lambda_1, \lambda_2, \mu)$ . (5 marks)
- iii) Find the values of  $x_1$  and  $x_2$ . (4 marks)

- b) Use the KKT conditions to find and classify the extrema for the following problem.  
For inequalities, rewrite them in the form  $g(x) \leq 0$  if necessary. (10 marks)

$$\begin{aligned} \text{Maximise} \quad & f(x, y) = x^2y \\ \text{subject to} \quad & x + y - 1 = 0, \\ & -x \leq 0. \end{aligned}$$

## Question 2 (20 marks)

An emergency response agency must strategically choose optimal distribution center locations from five candidate facilities ( $F_1, F_2, F_3, F_4$ , and  $F_5$ ) to distribute a single commodity (vaccines) to seven regional clinics ( $C_1$  through  $C_7$ ). Given the following information, formulate the optimisation model of the problem.

Table 1: Facility Transportation Costs, Capacities, and Fixed Costs

Facility	Transportation cost (\$)							Capacity (units)	Fixed cost (\$)
	C1	C2	C3	C4	C5	C6	C7		
F1	3	2	4	5	6	4	3	500	1200
F2	4	3	4	2	5	3	4	600	1400
F3	3	4	3	5	4	2	3	550	1300
F4	2	3	5	4	3	5	4	450	1000
F5	5	3	4	3	2	3	5	600	1500
<b>Demand</b>	200	150	180	170	130	140	160		

You are provided with the following notation for use in formulating the model:

- $K_j$  : capacity of facility  $j \in V_1 = \{F_1, F_2, \dots, F_5\}$
- $F_j$  : fixed cost to open facility  $j$
- $z_j \in \{0, 1\}$  : binary variable, which is 1 if facility  $j$  is opened, 0 otherwise
- $D_k$  : demand at clinic  $k \in V_2 = \{C_1, C_2, \dots, C_7\}$
- $y_{jk} \geq 0$  : quantity shipped from facility  $j$  to clinic  $k$
- $c_{jk}$  : transportation cost per unit from facility  $j$  to clinic  $k$

a) Define the decision variables required for this optimisation problem. (5 marks)

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- b) Formulate the objective function and briefly explain the objective and all the terms  
(9 marks)
- c) Write down all the constraints of the model, categorised as follows: (6 marks)
- i) Demand Satisfaction: Ensure that each clinic receives its required quantity.
  
  
  
  
  
  
  
  
  
  
  - ii) Facility Capacity: Ensure that no facility exceeds its shipping capacity, and shipments only occur from open facilities.
  
  
  
  
  
  
  
  
  
  
  - iii) Binary and Non-Negativity: Enforce binary status for facility openings and non-negativity for shipment quantities.

**Question 3 (20 points)**

A small logistics company is planning how to meet delivery demand for the next **2 months**. They are evaluating two options to cover vehicle capacity:

- **Option 1 – Rent on demand:** Pay for vans as needed from a daily rental company.
- **Option 2 – Monthly lease:** Lease 8 vans for 2 months, and rent extras when needed.

**Information:**

- Each van can handle **1,000 packages per month**.
  - Expected base demand: **8,000 packages/month**.
  - Demand may rise 20% or fall 20% with the probabilities 0.2 and 0.3, respectively, or remain the same with a probability of 0.5.
  - Rental cost: \$700 per van per month.
  - Lease cost: \$600 per van per month (2-month contract).
  - Revenue: \$1 per package.
  - Monthly discount rate:  $k = 10\%$
  - Assume demand is **independent across months**.
- (a) Draw a decision tree diagram to represent the uncertainty of the demand. Include all relevant information such as the demand for each node and probabilities. (5 marks)

- (b) Use the following tables to calculate revenue, costs, and profits for option 1 under different demand scenarios. (10 marks)

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<b>Period 2 Demand</b>	<b>Revenue</b>	<b>Cost</b>	<b>Profit</b>
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<b>Period 1 Demand</b>	<b>EMV</b>	<b>PV</b>	<b>Revenue</b>	<b>Cost</b>	<b>Profit</b>
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<b>Period 0 Demand</b>	<b>EMV</b>	<b>PV</b>	<b>Revenue</b>	<b>Cost</b>	<b>Profit</b>
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- (c) If the profit of option 2 is \$6430, which option should the company choose? (5 marks)

**Question 4 (20 points)**

You are the inventory analyst for a distribution centre that tracks weekly product demand. You are tasked with using two different forecasting methods to predict demand for the upcoming week and evaluate their performance.

<b>Week</b>	1	2	3	4	5	6	7
<b>Demand (Units)</b>	120	130	125	135	140	150	145

- (a) Fill in the following table and determine the regression equation using the least squares method and use the equation to forecast demand for Week 8. (8 marks)

<b>Week <math>X</math></b>	<b>Demand <math>Y</math></b>	$X^2$	$XY$	$\hat{Y}$	$ Y - \hat{Y} $
1					
2					
3					
4					
5					
6					
7					
$\Sigma$					

- (b) Using **Exponential Smoothing (SES)** with smoothing constant  $\alpha = 0.4$  and initial forecast of 120, fill in the following table and compute the forecast for Week 8. (8 marks)

Week $X$	Demand $Y$	$\hat{Y}$	$ Y - \hat{Y} $
1			
2			
3			
4			
5			
6			
7			
$\Sigma$			

- (c) Compute the **mean absolute error (MAE)** for Weeks 1–7 using linear regression and SES methods. Use the following formula for absolute error: (2 marks)

$$MAE = \frac{1}{n} \sum_{i=1}^n |\text{Actual Demand} - \text{Predicted Demand}|$$

- (d) Which method would you recommend for forecasting in this case, and why? (2 marks)



(b) What is the optimal order quantity for BookNest? (2 marks)

(c) What is the annual total cost under this policy? (2 marks)

(d) What is the cycle inventory maintained by BookNest under the optimal policy? (2 marks)

(e) How does the cycle inventory compare if the supplier instead charges a flat price of \$9.50 per book with no quantity discounts? (4 marks)