YANSHAN UNIVERSITY

PRACTICE EXAMINATIONS – June/July 2026

Unit Name: Supply Chain Modelling and Optimisation

Duration: 2 hours

Total Marks: 100

Calculator: Yes, any hand-held calculator approved by Yanshan University

THIS IS A CLOSED BOOK EXAM

IMPORTANT INFORMATION

Mobile phones or any other devices capable of communicating information are prohibited during examinations.

Electronic Organizers/PDAs (with the exception of calculators) or any other similar devices capable of storing restricted text or restricted information are prohibited during examinations.

Other Information: This paper contains **4** questions

Attempt as many questions as you can.

All working must be shown

Surname: ______________________________

Given Name: ______________________________

Student Number: ______________________________

Question 1 (25 marks)

Blue Computers, a PC manufacturer in the United States, currently operates two production plants located in Kentucky and Pennsylvania. The Kentucky plant has an annual production capacity of 1 million units, while the Pennsylvania plant has an annual capacity of 1.5 million units.

The company serves five regional markets: Northeast, Southeast, Midwest, South, and West. Each PC is sold at a price of $1,000 per unit. Blue Computers forecasts a 50% increase in demand across all regions in the coming year, after which demand is expected to either remain at the increased level or continue to grow, depending on market conditions. To accommodate future demand, the company plans to establish an additional plant with a production capacity of 1.5 million units per year.

Two potential locations are under consideration: North Carolina and California. The company pays a federal income tax of 20% and state/local taxes of 7% on income generated by each plant. As an incentive to attract the new facility, the state of North Carolina has offered to reduce state/local taxes from 7% to 2% for the first three years of operation.

Table 1 provides the annual fixed costs, unit production and shipping costs, and current regional demand before the anticipated demand increase.

Table 1: Production, transportation costs, and regional demand

Plant Location	Production and Shipping Cost ($/unit)					Annual Fixed Cost (million $)
	Northeast	Southeast	Midwest	South	West	
Kentucky	185	180	175	175	200	150
Pennsylvania	170	190	180	200	220	200
North Carolina	180	180	185	185	215	150
California	220	220	185	195	175	150
Demand (units/month)	700,000	400,000	400,000	300,000	600,000	

(a) Formulate a supply chain network design optimisation model that determines the optimal distribution of production among the existing plants and the proposed new facility. The objective is to minimise the total annual network cost. Clearly define:

(i) Sets and indices (0.5 marks)

(ii) Model parameters (0.5 marks)

(iii) Decision variable(s) (1 mark)

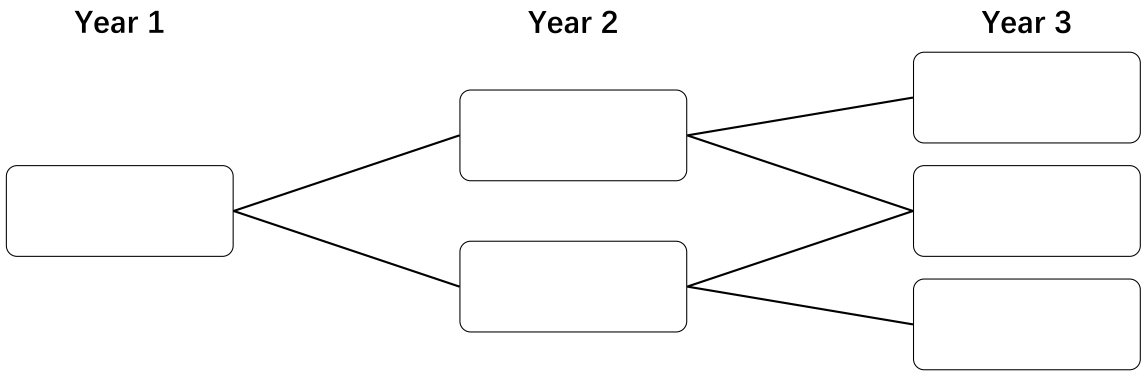
(iv) Objective function (3 marks)

(v) Constraints (4 marks)

(b) Construct a decision tree covering Years 1, 2 and 3, and determine whether the new plant should be located in North Carolina or California using a NPV analysis.

- At the beginning of each year, demand either increases by 50% or remains at its current level, with equal probability.
- Production, transportation, and fixed costs remain constant throughout the planning horizon.
- A discount rate of 10% is used for NPV calculations.

(i) Complete the decision tree by filling all the nodes and write down probability for each branch. (3 marks)



(ii) Complete the working tables for option 1, **a new plant at California.** (6 marks)

Table 2: Year 3 working table. Calculate in 10^6 and round to 2 decimal places.

Demand	Revenue	Fixed Cost + Tax	Profit	EV

Table 3: Year 2 working table. Calculate in 10^6 and round to 2 decimal places.

Demand	EMV	PV	Revenue	Fixed Cost + Tax	Profit	EV

Table 4: Year 1 working table. Calculate in 10^6 and round to 2 decimal places.

Demand	EMV	PV	Revenue	Fixed Cost + Tax	Profit	NPV

(ii) Complete the working tables for option 2, **a new plant at North Carolina.** (6 marks)

Table 5: Year 3 working table. Calculate in 10^6 and round to 2 decimal places.

Demand	Revenue	Fixed Cost + Tax	Profit	EV

Table 6: Year 2 working table. Calculate in 10^6 and round to 2 decimal places.

Demand	EMV	PV	Revenue	Fixed Cost + Tax	Profit	EV

Table 7: Year 1 working table. Calculate in 10^6 and round to 2 decimal places.

Demand	EMV	PV	Revenue	Fixed Cost + Tax	Profit	NPV

(iii) Which location should Blue PC build the new plant? (1 mark)

Question 2 (25 marks)

Harvey Norway sells three sizes of beds: King size, Queen size, and Single size. Annual demands for the three-size products are $D_K = 1,000$ for the king-size beds, $D_Q = 500$ units for the queen-size beds, and $D_S = 100$ units for the single-size bed, and each model costs Harvey $1000, $800 and $400, respectively. A fixed transportation cost of $500 is incurred each time an order is delivered. For each model ordered and delivered on the same truck, an additional fixed cost of $150 is incurred for receiving and storage. Harvey Norway incurs a holding cost of 20 percent. Evaluate optimal lot sizes, order frequency, and annual inventory-related cost assuming

(a) Lots for each product are ordered and delivered independently. (10 marks)

(b) Lots are delivered jointly for a selected subset of the products. (15 marks)

Question 3 (25 marks)

A retailer of electronic accessories sells a popular wireless headset. Historical sales data indicate an annual demand of 12,000 units. The cost of placing an order is $75 per order, and the annual holding cost is $1.20 per unit. The retailer operates 300 days per year.

(a) Assuming deterministic demand and a constant lead time of 4 days:

(i) Determine the Economic Order Quantity (EOQ). (3 marks)

(ii) Calculate the total annual inventory cost. (5 marks)

(iii) Determine the reorder point. (2 marks)

(b) Suppose daily demand follows a normal distribution with a mean of 40 units per day and a standard deviation of 8 units per day. Assuming a constant lead time of 4 days, determine the safety stock and reorder point for a cycle service level of 85%. (6 marks)

(c) Assume that both demand and lead time are uncertain. Daily demand is normally distributed with mean 40 units and standard deviation 8 units, and lead time is normally distributed with mean 4 days and standard deviation 1.5 days. Determine the safety stock and reorder point for a cycle service level of 99.5%. (9 marks)

Question 4 (25 marks)

A newspaper is attempting to forecast future subscriptions to the Sunday paper. The numbers of subscriptions for the past three weeks are:

Week	Sunday Subscriptions (000)
1	452
2	396
3	402

(a) Compute a three-week moving average forecast for the number of subscriptions in Week 4. (5 marks)

(b) Compute a weighted three-week moving average forecast for the number of subscriptions in Week 4 assuming weights of 0.75, 0.20, and 0.05. Assign the weights to the logical week. (5 marks)

(c) Compute the forecast for Week 4 using exponential smoothing assuming that

$$\alpha = 0.2,$$

and using the average of Weeks 1 and 2 as the forecast for Week 3. (7 marks)

(d) What changes would you notice in the variability of forecast numbers in changing from a three-week moving average to a six-week moving average? Which type of moving average forecast responds more quickly to large swings in demand? (4 marks)

(e) For exponential smoothing forecasts, what does α signify? Would a higher or lower value of α result in more stable forecasts? (4 marks)

END OF PRACTICE EXAMINATION

FORMULA SHEET

Network Design Under Uncertainty

$$PV_t = \frac{C_t}{(1+r)^t}, \quad NPV = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

$$EMV = \sum (\text{Probability} \times \text{Payoff})$$

Forecasting Methods

- **Naive:** $F_{t+1} = F_t$
- **Moving Average (n-MA):**
  $\hat{Y}_{t+1} = \frac{Y_t + \cdots + Y_{t-n+1}}{n}$
- **Weighted Moving Average (n-WMA):**
  $\hat{Y}_{t+1} = w_1 Y_t + \cdots + w_n Y_{t-n+1}$
  $\sum w_i = 1$
- **Simple Exponential Smoothing:**
  $\hat{Y}_{t+1} = \alpha Y_t + (1-\alpha)\hat{Y}_t$
- **Linear Regression**

$$\hat{y} = a + bx, \qquad b = \frac{\Delta y}{\Delta x}$$

$$b = \frac{\sum XY - n\bar{x}\bar{y}}{\sum X^2 - n\bar{x}^2} = \frac{\sum (X_i - \bar{x})(Y_i - \bar{y})}{\sum (X_i - \bar{x})^2}$$

$$a = \bar{y} - b\bar{x}, \qquad \bar{x} = \frac{\sum X}{n}, \qquad \bar{y} = \frac{\sum Y}{n}$$

- **Multiple Regression (2 indep variables**

$$\hat{y} = a + b_1 x_1 + b_2 x_2$$

$$\sum Y = na + b_1 \sum X_1 + b_2 \sum X_2$$

$$\sum X_1 Y = a \sum X_1 + b_1 \sum X_1^2 + b_2 \sum X_1 X_2$$

$$\sum X_2 Y = a \sum X_2 + b_1 \sum X_1 X_2 + b_2 \sum X_2^2$$

- **Forecasting Metrics**

$$SSE = \sum (y_i - \hat{y}_i)^2, \quad SAE = \sum |y_i - \hat{y}_i|$$

$$MAE = \frac{1}{n} \sum |y_i - \hat{y}_i|, \quad RMSE = \sqrt{\frac{1}{n} SSE}$$

$$MAPE = \frac{100\%}{n} \sum \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

Economic Order Quantity (EOQ)

$$Q^* = \sqrt{\frac{2DA}{h}}, \quad OC = \frac{DA}{Q^*}, \quad IHC = \frac{Q^* h}{2}$$

$$TC = \frac{DA}{Q^*} + \frac{Q^* h}{2} = \sqrt{2DAh}$$

$$T = \frac{Q^*}{D}, \quad \text{Order Frequency} = \frac{D}{Q^*}$$

$$\frac{TC(Q_1)}{TC(Q^*)} = \frac{1+b^2}{2b}$$

EOQ with Shortage

$$Q^* = \sqrt{\frac{2DA}{h} \frac{k+h}{k}}, \quad S^* = Q^* \frac{h}{k+h}$$

$$Q^* - S^* = Q^* \frac{k}{k+h}$$

$$IHC = \frac{(Q^* - S^*)^2}{2Q^*} h, \quad SC = \frac{(S^*)^2}{2Q^*} k$$

$$TC^* = \frac{DA}{Q^*} + \frac{(Q^* - S^*)^2}{2Q^*} h + \frac{(S^*)^2}{2Q^*} k$$

Economic Production Quantity (EPQ)

$$Q^* = \sqrt{\frac{2DA}{h} \frac{r}{r-D}} = \sqrt{\frac{2DA}{h(1-D/r)}}$$

$$T = \frac{Q^*}{D} = T_r + T_d$$

$$T_r = \frac{Q^*}{r}, \qquad T_d = \frac{Q^*}{D}\left(1 - \frac{D}{r}\right)$$

$$I_{\text{max}} = Q^* \left(1 - \frac{D}{r}\right)$$

$$IHC = \left(1 - \frac{D}{r}\right) \frac{Q^*}{2} h$$

$$TC^* = \frac{DA}{Q^*} + \left(1 - \frac{D}{r}\right) \frac{Q^*}{2} h$$

EOQ with All-Units Discount

$$Q_i^* = \sqrt{\frac{2DA}{IC_i}}$$

$$\hat{Q}_i = \begin{cases} q_i, & Q_i^* < q_i \\ Q_i^*, & q_i \le Q_i^* \le q_{i+1} \\ q_{i+1} - 1, & Q_i^* > q_{i+1} \end{cases}$$

$$TC_i(Q) = C_i D + \frac{D}{Q} A + \frac{Q}{2} h_i$$

$$h_i = IC_i, \qquad Q^* = \arg\min_i TC_i(\hat{Q}_i)$$

EOQ with Incremental Discount

$$Q_j^* = \sqrt{\frac{2(R_j - C_j q_j + A)d}{IC_j}}$$

$$R_j = C_1(q_2 - q_1) + \cdots + C_{j-1}(q_j - q_{j-1})$$

$$C(Q) = R_j + C_j(Q - q_j)$$

$$TC(Q_j) = \frac{C(Q_j)}{Q_j} d + \frac{D}{Q_j} A + \frac{Q_j}{2}\left(I \frac{C(Q_j)}{Q_j}\right)$$

Multi-Commodity Inventory

(i) Aggregate into single order

$$T^* = \sqrt{\frac{2(A + \sum_{i=1}^n A_i)}{\sum_{i=1}^n D_i h_i}}, \quad Q_i^* = D_i T^*$$

$$TC^* = \sqrt{2\left(A + \sum_{i=1}^n A_i\right) \sum_{i=1}^n D_i h_i}$$

(ii) Delivered independently

$$Q_i^* = \sqrt{\frac{2DA}{h_i}}$$

$$TC^* = \sum_{i=1}^n \left(\frac{D}{Q_i^*} A + \frac{Q_i^*}{2} h_i\right) = \sum_{i=1}^n \sqrt{2DAh_i}$$

(iii) Delivered Jointly for All Products

$$n^* = \sqrt{\frac{\sum_{i=1}^k D_i IC_i}{2A^*}}, \quad A^* = A + \sum_{i=1}^n A_i$$

$$Q_i^* = \frac{D_i}{n^*}, \; OC = n^* A^*, \; IHC = \frac{1}{2n^*} \sum_{i=1}^n D_i IC_i$$

$$TC^* = n^* A^* + \frac{1}{2n^*} \sum_{i=1}^n D_i IC_i$$

(iv) Selected Subset Joint Delivery

$$\bar{n}_i = \sqrt{\frac{IC_i D_i}{2(A + A_i)}}, \quad \bar{n} = \max\{\bar{n}_i\},$$

$$\bar{\bar{n}}_i = \sqrt{\frac{IC_i D_i}{2A_i}}, \quad \bar{m}_i = \frac{\bar{n}}{\bar{\bar{n}}_i}, \quad m_i = \lceil \bar{m}_i \rceil$$

$$n = \sqrt{\frac{\sum IC_i m_i D_i}{2\left[A + \sum (A_i/m_i)\right]}}, \quad n_i = \frac{n}{m_i}$$

$$Q_i^* = D_i / n_i$$

$$TC = n\left[A + \sum_i \frac{A_i}{m_i}\right] + \frac{1}{2n} \sum_i IC_i D_i m_i$$

Continuous Review (Q,R)

$$R = (\text{demand during } L) + SS$$

- **Both constant:** $R = dL$
- **Variable** d: $R = \bar{d} + z\sigma_d\sqrt{L}$
- **Variable** L: $R = d\bar{L} + zd\sigma_{LT}$
- **Both variable:** $R = \bar{d}\bar{L} + z\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$

Periodic Review (P)

$$TIL = (\text{demand during } P + L) + SS$$

- **Both constant:** $TIL = d(P + L)$
- **Variable** d: $TIL = \bar{d}(P+L) + z\sigma_d\sqrt{P + L}$
- **Variable** L: $TIL = d(P + \bar{L}) + zd\sigma_{LT}$
- **Both variable:**

$$TIL = \bar{d}(P + \bar{L}) + z\sqrt{\sigma_d^2(P + \bar{L}) + \bar{d}^2\sigma_{LT}^2}$$

$$T = \frac{Q^*}{\bar{d}} = \sqrt{\frac{2K}{\bar{d}h}}, \quad I_{\text{max}} = TIL$$

Newsvendor (Single-Period)

$$C_o = c - u, \quad C_u = r - c$$

$$CR = \frac{C_u}{C_u + C_o} = \frac{r - c}{r - u}$$

$$P(D \le Q^*) = CR, \quad Q^* = \mu + z\sigma$$

$$\Phi(z) = CR$$

$$E[TC(Q)] = C_o E[(Q-D)^+] + C_u E[(D-Q)^+]$$

$$E[(Q - D)^+] = \sigma[\phi(z) + z\Phi(z)]$$

$$E[(D - Q)^+] = \sigma[\phi(z) - z(1 - \Phi(z))]$$

$$z = \frac{Q - \mu}{\sigma}$$

Table 8: Standard Normal Distribution Table ($\Phi(z)$)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9989	0.9990